Lecture 12 — 10/18

New HW - somewhat different ...

read / understand papers

R is typically defined as the radius where the interaction strength equals the linewidth of the excitation transition.

A few review articles:

Saffman, Walker, Mølmer, RMP **82**, 23123 (2010)

Adams, Pritchard, Shaffer, J. Phys. B (AMO) **53**, 012002 (2020)

Dunning, Killian, www.scientia.global/rydberg-atoms-giants-of-the-atomic-world/

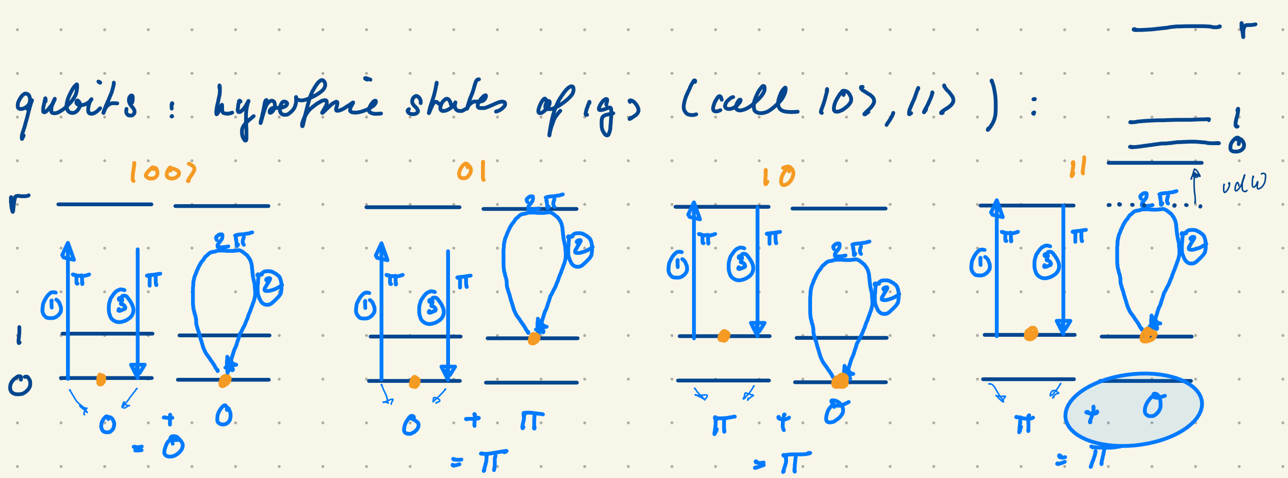
Gate using Rydberg blockade:

phase gate:
$$\begin{array}{c|cccc} & 00 & 01 & 10 & 11 \\ \hline 00 & a & & & \\ 01 & & b & & \\ 10 & & & c & \\ 11 & & & & d \end{array} \quad \underbrace{\begin{pmatrix} a & & & 0 \\ & b & & \\ & & c & \\ 0 & & & d \end{pmatrix}}_{U}$$

U is a nonlinear gate if $|a| = |b| = |c| = |d| = 1$ and normalize to $a = 1$

$\Rightarrow \arg b + \arg c \neq \arg d$

qubits: hypefine states of $|g\rangle$ (call $|0\rangle, |1\rangle$):



$$U = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

↑π : π-pulse 2π : 2π pulse

One 2π-pulse on a transition (or a combination of 2 π-pulses) gives an overall shift of π. The same pulse not on a transition does nothing.

NB: Why?

$$H = \begin{pmatrix} -\Delta & -\frac{\Omega}{2}e^{i\varphi} \\ -\frac{\Omega}{2}e^{-i\varphi} & \Delta \end{pmatrix} \quad \text{for } \Omega_2 = \Omega e^{i\varphi}$$

Simplify: no detuning $\Delta = 0$
real Ω_2 $\Rightarrow$ $H = \begin{pmatrix} 0 & -\frac{\Omega}{2} \\ -\frac{\Omega}{2} & 0 \end{pmatrix}$

$\Rightarrow$ with $|\psi(0)\rangle = |0\rangle$

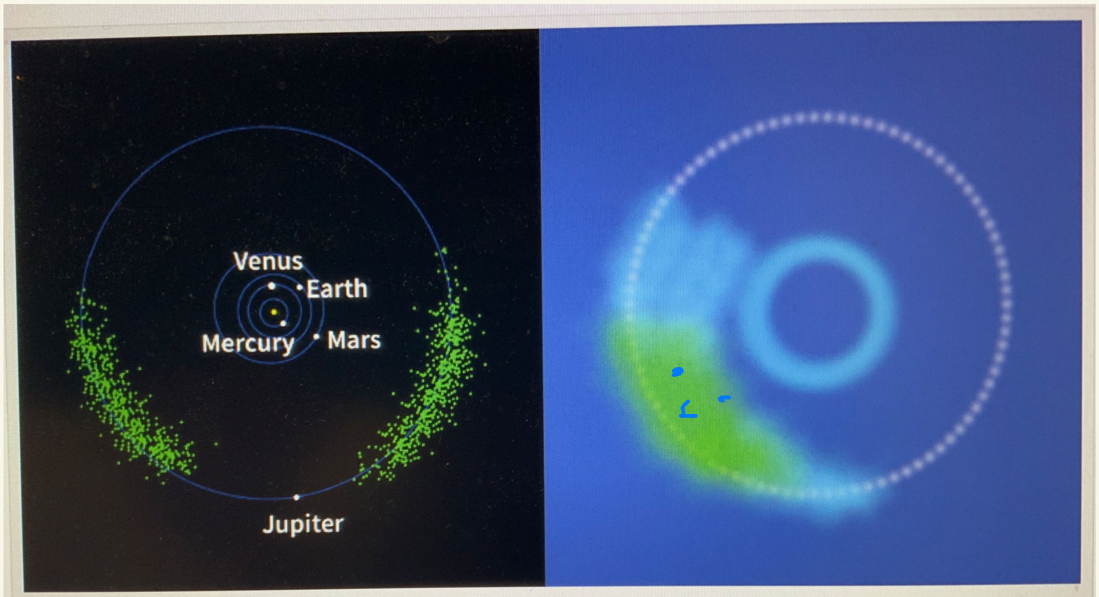
$$|\psi(t)\rangle = \cos\frac{\Omega t}{2}|0\rangle + i\sin\frac{\Omega t}{2}|1\rangle$$

$\Rightarrow \pi = \Omega t \Rightarrow |\psi(t)\rangle = i|1\rangle$

$2\pi = \Omega t \Rightarrow |\psi(t)\rangle = -|0\rangle$

Other applications:

- Controlling e^--motion: orbital period long (~ 10's μs)
  use external field kicks ⇒ any form of orbit



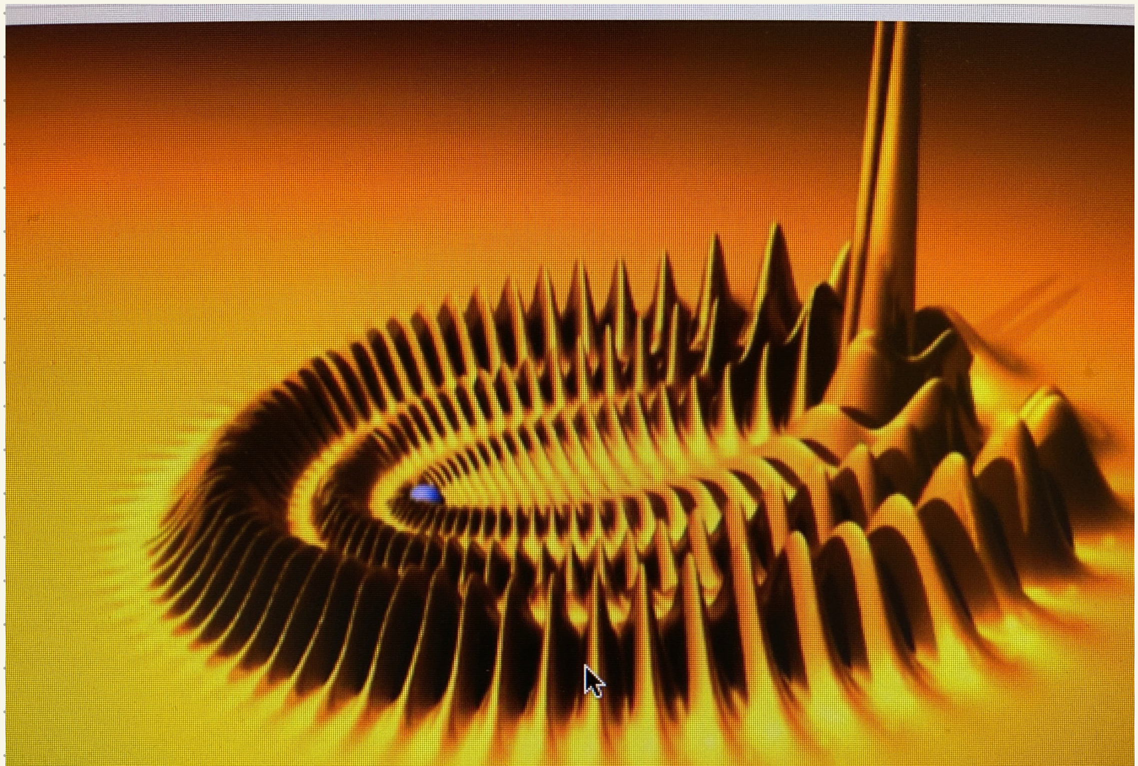
The image on the left shows Jupiter's Trojan asteroids in their orbit, and the image on the right shows the electron probability density distribution in a Rydberg atom that mimics their behaviour

Example: mimic Jupiter's Trojan asteroids

- Sensing: use EIT to detect in/out of resonance

explain EIT?

- Rydberg molecules:



Theoretical electron density pattern of a weakly bound Rydberg molecule, resembling a trilobite. CREDIT: Chris Greene, Purdue University

Rydberg + g.s. atom

⇒ sense "trilobite"-formed field / wavefct of Rydberg

e) Dirac Equation

(Relativistic "corrections")

(i) Schr. Eq. violates relativity!

energy: (free particle)

$$E^2 = m^2c^4 + p^2c^2$$

with field:

$$E \longrightarrow E - q\varphi$$

$$\vec{p} \longrightarrow \vec{p} - q\vec{A}$$

$E \to i\hbar\partial_t$; $\vec{p} = -i\hbar\vec{\nabla}$ (*)

$\Rightarrow$ <u>Klein – Gordon Eq.</u>

$$\left(\frac{1}{c^2}\partial_t^2 - \nabla^2\right)\phi + \frac{m^2c^2}{\hbar^2}\phi = 0$$

$\oplus \rightarrow$ plane wave solution ☺

$\ominus \rightarrow$ $\underbrace{\frac{d}{dt}\int_{space}\phi^*\phi\, d^3x}_{\overset{!}{=} \text{ normalized (constant)}} = \underbrace{\int(\phi^{*\prime}\phi + \phi^*\phi')dx^2}_{\text{not necessarily } = 0}$ ☹

b/c it is <u>second</u> order diff. eq. $\Rightarrow$ <u>free choice</u> for both initial value of ϕ and ϕ'.

(but does work for spin-0 particles)

- <u>Dirac</u> wanted linear in E (and ∂_t)

"square root" of Klein-Gordon

$\Rightarrow$ (at least) 2 components

("spinor")

$$\boxed{H = c\vec{\alpha}\cdot\hat{\vec{p}} + \beta mc^2 + V(r)} \quad (1a)$$

$(H|\psi\rangle = E|\psi\rangle)$ ← (*)

with $E^2 = \langle H\rangle^2 \overset{!}{=} m^2c^4 + p^2c^2$

$$\Rightarrow \left.\begin{aligned} \alpha_i\alpha_j + \alpha_j\alpha_i &\overset{!}{=} 2\delta_{ij}\mathbb{1} \\ \alpha_i\beta + \beta\alpha_i &= 0 \\ \beta^2 &= \mathbb{1} \end{aligned}\right\} \quad (1b)$$

covariant form:

$\gamma_0 = \beta$

$\gamma_i = \alpha_i$

$\Rightarrow i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0$

choose simplest representation:

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \; ; \quad \beta = \begin{pmatrix} 1\!\!1_{2\times2} & 0 \\ 0 & -1\!\!1_{2\times2} \end{pmatrix}$$

(1b needs at least 4 dof to be solved)

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} \vec{\psi}_+ \\ \vec{\psi}_- \end{pmatrix} ; \; \vec{\psi}_\pm \text{ '2D spinors'}$$

$$\left.\begin{aligned} (1a) \quad (\vec{\sigma}\cdot\vec{p})\,\vec{\psi}_+ &= \tfrac{1}{c}\left(E + mc^2 - V(r)\right)\vec{\psi}_- \\ (\vec{\sigma}\cdot\vec{p})\,\vec{\psi}_- &= \tfrac{1}{c}\left(E - mc^2 - V(r)\right)\vec{\psi}_+ \end{aligned}\right\} \quad (1a^*)$$

non-relativistic: $|\vec{\sigma}\cdot\vec{p}| \approx mv \ll mc^2$

<u>$E>0$</u>: $|E + mc^2 - V| \gg |E - mc^2 - V|$

$$|\vec{\sigma}\cdot\vec{p}| \propto mv, \quad E - mc^2 - V \approx \tfrac{1}{2}mv^2$$

$$(1a^*) \Rightarrow \frac{|\vec{\psi}_+|}{|\vec{\psi}_-|} \propto \frac{2c}{v} \gg 1$$

$\Rightarrow \vec{\psi}_+$ (for $E>0$) more relevant!

(opposite for $E<0$: $|\psi_-| \gg |\psi_+|$)

$\Rightarrow$ "positive" ($\vec{\psi}_+$) and "negative" ($\vec{\psi}_-$) energy solutions

(Interpretation: "electron sea": $E<0$ "holes")

$(1a^*) \Rightarrow$

$$(\vec{\sigma}\cdot\vec{p})\frac{c^2}{E + mc^2 - V}(\vec{\sigma}\cdot\vec{p})\,\vec{\psi}_+ = (E - mc^2 - V)\,\vec{\psi}_+$$

$(\varepsilon \equiv E - mc^2)$

- $\frac{c^2}{2mc^2+\varepsilon-V} \approx \frac{1}{2m}\left(1-\frac{\varepsilon-V}{2mc^2}\right)$

- $(\vec{\sigma}\cdot\vec{A})(\vec{\sigma}\cdot\vec{B}) \equiv \vec{A}\cdot\vec{B} + i\vec{\sigma}(\vec{A}\times\vec{B})$

$\Rightarrow (\vec{\sigma}\cdot\vec{p})\, f(r)\, (\vec{\sigma}\cdot\vec{p}) =$

$= f(r)(\vec{\sigma}\cdot\vec{p})(\vec{\sigma}\cdot\vec{p}) - i\hbar \frac{f'(r)}{r}(\vec{\sigma}\cdot\vec{r})(\vec{\sigma}\cdot\vec{p}) =$

$= f(r)\vec{p}^2 - i\hbar\frac{f'(r)}{r}\left(\vec{r}\cdot\vec{p} + i\vec{\sigma}(\vec{r}\times\vec{p})\right)$

with $f(r) = \frac{1}{2m}\left(1-\frac{\varepsilon - V(r)}{2mc^2}\right)$

$$\boxed{\hbar\vec{\sigma} \equiv 2\vec{S}\ ;\ \vec{r}\times\vec{p} = \vec{L}}$$

definition of quantity with dimension of $\hbar$ (ang. mom.)

$\Rightarrow$ call "spin"

$\vec{s}$: single e^-
$\vec{S}$: all e^-

Why factor of 2?

↳ such that $\vec{s}$ spin commutation relations can be satisfied

↳ there are (at least) 2 spin components $\Rightarrow$

$$2s+1 \overset{!}{=} 2$$

$$\left[\left(1-\frac{\varepsilon-V(r)}{2mc^2}\right)\frac{p^2}{2m} - i\hbar\frac{1}{4m^2c^2}\frac{V'(r)}{r}(\vec{r}\cdot\vec{p}) + \frac{1}{2m^2c^2}\frac{V'(r)}{r}\vec{S}\cdot\vec{L}\right]\psi_+ = (\varepsilon - V(r))\psi_+$$

with $\varepsilon - V(r) \approx \frac{p^2}{2m}$

higher spins?

$\Rightarrow \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}$ for higher-dim. Paulis