Lecture 14 — 10/25

- Reminder: please submit HW!

Topics: "all the multi-level" stuff...:
- hyperfine interaction
- atoms in fields:
  - → magnetic (i.e. interaction with magn dipoles)
  - → electric (change of transitions)

g) Effects of nucleus

Real nuclei:

Have finite mass ($M \neq m_n$)

angular momentum ($\vec{I}$)

finite volume (of charge)

↳ dipole mom. (magn)

quadr. " (el.)

...

Effects:

	result	observe via
(i) finite mass	"mass shift"	isotope shift
(ii) finite volume	"volume shift"	— " —
(iii) magn. dipole	"hyperfine struct"	hf splitting
(iv) el. quadrupole	"	"
⋮		

(iii) + (iv) $\Rightarrow$ $\vec{F} = \vec{J} + \vec{I}$ total atomic ang. momentum

Hyperfine interaction

(Budker, Kimball, DeMille: 1.4 Hf + Zeeman in Hydrogen)

$$\boxed{H_{hf} = -\vec{\mu}_I \cdot \vec{B}_J = a h \vec{I} \cdot \vec{J}}$$

general remarks:

→ hf splitting: very small

$\sim$ fine structure $\mathcal{O}(\alpha^2) \cdot \frac{m_e}{m_n}$

→ very good (meta-) stable ground states (e.g. EIT, qubits)

→ nucleus $\hat{=}$ charge cloud with angular mom.

⇒ dipole, quadrupole, ... moments : magnetic + electric (because of parity & time reversal): only <u>magn.</u> dipole, only <u>el.</u> quadr., ...

↳ mgn. dip: $I \geq \frac{1}{2}$, splitting only for $J \geq \frac{1}{2}$ (el. quadr: $I, J \geq 1$)

<u>Quantities</u>

$\vec{\mu}_s = -g_s \mu_0 \frac{\vec{S}}{\hbar}$ electron spin mgn. moment

$\vec{\mu}_\ell = -g_\ell \mu_0 \vec{L}/\hbar$ orbital

$\vec{\mu}_I = g_I \mu_N \frac{\vec{I}}{\hbar}$

g : "gyromagnetic ratio"
"Landé g-factor"

$g_s = 2$ ⟶ Dirac

$g_\ell = 1$ (= classical formula: μ = current × area)

$g_p = 5.586$, $g_n = -3.826$
proton neutron

μ_0 ("Bohr magneton") $= \frac{e\hbar}{2m_e} \approx 9.24 \cdot 10^{-24} \frac{J}{T}$

μ_N (nuclear magneton) $= \frac{e\hbar}{2m_n} \approx 5.05 \cdot 10^{-27} \frac{J}{T}$

in general: $\vec{\mu}_F = \vec{\mu}_I + \vec{\mu}_J$

$\vec{\mu}_J = g_J \frac{q}{2m} \vec{J}$, $\vec{\mu}_I = ...$

$\vec{F} = \vec{J} + \vec{I}$ ⇒ F, M_F good q. numbers

$$H_{hf} = -\vec{\mu}_I \cdot \vec{B}_J = a_h \, \vec{I}\cdot\vec{J}$$

$\vec{\mu}_I$: magn. moment of nucleus

$\vec{B}_J$: magn. field due to magn. moment of electrons

$$\vec{B}_J \propto \vec{J} \implies \vec{I}\cdot\vec{B}_J = \frac{(\vec{I}\cdot\vec{J})(\vec{J}\cdot\vec{B}_J)}{\vec{J}^2}$$

$$\implies \boxed{a_h = -\frac{\mu_I}{I}\cdot\frac{\vec{J}\cdot\vec{B}_J}{\vec{J}^2}}$$

$$\vec{J} = \vec{L} + \vec{S} \qquad \vec{B}_J = \vec{B}_L + \vec{B}_S$$

$\vec{B}_L$: from e^- current

$\vec{B}_S$: magn. sphere

Electric quadrupole:

Quadrup. $Q = \frac{1}{e}\int d^3r \, \rho(\vec{r}) \, (3z^2 - r^2)$

$$Q \begin{cases} < 0 & \text{"oblate"} \\ > 0 & \text{"prolate"} \end{cases}$$

$$E_{hf}^{Q} \propto \frac{Q}{a_B^2}\left\langle\frac{1}{r^3}\right\rangle \text{ small}: \quad Q \approx \langle r_{nucl}^2\rangle \approx 10^{-24}\,\mathrm{cm}^2$$

magnitude of hyperfine shift:

$$\frac{E_{hf}}{E_{fs}} \approx g_I \frac{m_e}{m_n}\frac{1}{Z} \approx 10^{-3} - 10^{-4}, \qquad \frac{E_{hf}}{\hbar} \propto \frac{1}{(\ell+3/4)^3}\,\mathrm{GHz}$$

Isotope effects

. Mass effect:

single e^-: $E_n = E_{n,0}\dfrac{m_n}{m_n + m_e} \approx E_{n,0}\left(1 - \underbrace{\dfrac{m_e}{m_n}}_{\Delta E}\right)$

$\geq 2e^-$: $\Delta E_{n.m_n} = -\frac{P^2}{2m_n}$: kinetic energy of nucleus around center-of-mass

$\vec{P} = \sum_{\text{all } e^-} \vec{p}_i$

$$= -\frac{m_e}{m_n}\left(\frac{1}{2m_e}\sum_i \vec{p}_i^{\,2} + \frac{1}{2m_e}\sum_{i\neq j}\vec{p}_i\cdot\vec{p}_j\right)$$

"normal shift" "specific shift" (= 0 for single e^-)

$$\Delta E_{specific} = (1-2S)\,\frac{m_e}{m_n}\,\frac{3f_{ns,n'l}}{2}\,\hbar\,\omega_{ns,n'l}$$

$\Rightarrow$ ΔE_{spec} has different sign for singlet ($S=0$) and triplet ($S=1$)

$3f_{ns,n'l}$: "oscillator strength" between states ns, $n'l$

$\left(f_{eg} = \frac{2m_e}{3\hbar}\sum |\langle e|r|g\rangle|^2\, \omega_{eg} \right)$

over all hyperfine states

$\Rightarrow$ $\Delta E_{spec} = 0$ if transition not allowed

$$\Rightarrow \boxed{\frac{\Delta E_{n,m_n+\Delta m_n} - \Delta E_{n,m_n}}{E_n} \approx \frac{m_e}{m_n}\,\frac{\Delta m_n}{m_n}}$$

isotope shift

Volume effect:

inside nucleus: potential less deep

$\Rightarrow$ valence e^- with non-zero density inside nucleus sees increased energy

$\Rightarrow$ transition energy is lower if effect is stronger for lower state.

Depends on $\frac{d\delta}{dn}$ (q. defect), Z, Z_{nucl}, $r_{nucleus}$

(Reference for this section: Sobel'man: Atomic Spectra and radiative transitions)

5) Atoms in DC fields

a) Magnetic fields

- classical circulating charge

interaction: $U = -\underbrace{\vec{\mu}}_{\text{magnetic moment}} \cdot \vec{B}$ (5.1)

angular momentum: $\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$

$\Rightarrow$ magn. moment: $\vec{\mu} = \frac{1}{2} \vec{r} \times \underbrace{\vec{i}}_{\text{current}} = \frac{q}{2}(\vec{r} \times \vec{v})$

$$\vec{\mu} = \frac{q}{2m} \vec{L} \equiv \gamma_e \vec{L}$$

"gyromagn. ratio"

$$\mu_0 = \frac{e\hbar}{2m_e} = 9.27408 \frac{J}{T}$$

"Bohr magneton"

Note: $\gamma_e = \frac{q}{2m}$ is general result for material with uniform charge-to-mass ratio.

- intrinsic electronic spin:

Dirac formalism: $\frac{|\vec{S}|}{\hbar} = \frac{1}{2}$

but: $\vec{\mu}_S = -g_e \mu_0 \frac{\vec{S}}{\hbar}$ with $g_e = 2$

"Landé g-factor"

$\Rightarrow e^-$ does not have uniform charge-to-mass ratio!

(really: $\frac{g_e}{2} = 1.00115965218 69 (14)$ due to QED corrections)

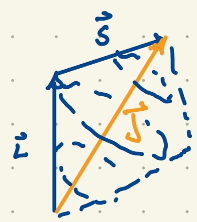
$g/2 = 1 + \frac{1}{2\pi}\alpha + \mathcal{O}(\alpha^2)$

- Landé g-factors

$\vec{J} = \vec{L} + \vec{S}$ gives good q. numbers (Dirac)

(for zero or low magnetic field)

→ g_S, g_L don't contribute independently!



(only $|\vec{L}|$, $|\vec{S}|$, L_z, S_z defined. For zero mgn. field: $\vec{J} \parallel$ quantization axis $\hat{=} \hat{z}$)

⇒ find sum of projections of $\vec{\mu}_S$ and $\vec{\mu}_L$ on $\vec{J}$.

$$\vec{L}: \mu_{L,J} = -\frac{\mu_0 |\vec{L}|}{\hbar} \frac{\vec{L}\cdot\vec{J}}{|\vec{L}|\cdot|\vec{J}|} \qquad \vec{S}: \mu_{S,J} = -g_s \frac{\mu_0 |\vec{S}|}{\hbar} \frac{\vec{S}\cdot\vec{J}}{|\vec{S}||\vec{J}|}$$

$$\vec{\mu}_J = -g_J \mu_0 \frac{\vec{J}}{\hbar} \qquad g_J = -\frac{\hbar}{\mu_0} \frac{\mu_{L,J} + \mu_{S,J}}{|\vec{J}|}$$

$$g_J = \frac{g_L \vec{L}\cdot(\vec{L}+\vec{S}) + g_s \vec{S}\cdot(\vec{L}+\vec{S})}{|\vec{J}|^2} =$$

Note: $\vec{L}\cdot\vec{S} = \frac{1}{2}(\vec{J}^2 - \vec{L}^2 - \vec{S}^2)$

$$= 1 + \frac{j(j+1) + s(s+1) - \ell(\ell+1)}{2j(j+1)}$$

(ignore $\vec{I}$: $U = -\vec{\mu}_J \cdot \vec{B}$,
$\vec{F} = \vec{I} + \vec{J}$: same logic)

Transitions: level j' ↔ level j''

three or more lines: "Zeeman"
(low magnetic fields)

e.g. $\Delta m (\equiv m_{j'} - m_{j''}) = -1$

$$\boxed{\Delta E = (g_{j'} m_{j'} - g_{j''} m_{j''}) \mu_0 B}$$ (from 5.1)

ΔE: change of transition frequency