New physics with AMO

→ lasers
→ cavity QED
→ ultracold temperatures
(cooling, trapping, controlling, manipulating ...)
→ (ultrashort) pulses

in order to
- find fundamental constants
- do metrology (clocks)
- q. information science
  (e.g. Rydberg arrays,
  ion traps
  molecules in tweezers
  q. gas microscope
  ....)
- material engineering

Housekeeping

- This is ARO I.
- Canvas: (show)
  - lecture notes (handwritten) posted
  - syllabus
  - homework
    → one HW – lecture notes (show overleaf)

  - student meetings → set up sheet on canvas
- Class

  - let me know about handwriting / legibility + pronunciation / intelligible

- grades:
  no exams
  HW + one lec : 60%
  final project : 30%
  participation : 10%
  - in class

  - during final present's
  - organize student meetings
  - ...

- presentations (towards end)
  → topic of your choice (but <u>not</u> own research
  → try something new

Questions ?

Modern Atomic Physics

(285A)

1. Units & Basic Quantities

atoms are very basic – what can they depend on?

a) Non-relativistic atom units should be dependent on

m : electron mass (if $m_e \ll m_p$)

e : elementary charge $\left(+\frac{1}{4\pi\epsilon_0}\right)$

$\hbar$: Planck's constant

NB: SI Coulomb's law:

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} = \underbrace{\frac{q_1}{\sqrt{4\pi\epsilon_0}}}_{\equiv \tilde{q}_1}\underbrace{\frac{q_2}{\sqrt{4\pi\epsilon_0}}}_{\equiv \tilde{q}_2}\frac{1}{r^2}$$

Dimension:

$$\frac{[M][L]}{[T]^2} = \frac{[\tilde{Q}]^2}{[L]^2}$$

$$\Rightarrow [\tilde{Q}] = \frac{\sqrt{[M][L]^3}}{[T]}$$

with: $[M]$ – dimension of mass

[L] - dimension of length
[T] - dimension of time
[Q] - dimension of charge

- Is there a basic dimensionless scale? (w/ $m, \tilde{e}, \hbar$) you know? how to find?

$$m^k \tilde{e}^l \hbar^m = [M]^k \left(\frac{[M]^{1/2} [L]^{3/2}}{[T]} \right)^l \cdot \left(\frac{[M][L]^2}{[T]} \right)^m$$

$$= [M]^{k+\frac{l}{2}+m} [L]^{\frac{3l}{2}+2m} [T]^{-l-m}$$

dimensionless:

$$\left.\begin{aligned} k+\frac{l}{2}+m &= 0 \\ \frac{3l}{2}+2m &= 0 \\ l+m &= 0 \end{aligned}\right\} l=0,\ m=0,\ k=0$$

$\Rightarrow$ **no dimensionless scale!**

- basic length scale?

ACT 1

$$[M]^{k+\frac{l}{2}+m} [L]^{\frac{3l}{2}+2m} [T]^{-l-m}$$

$$= [L]^p$$

$$\left.\begin{array}{l} k + \frac{l}{2} + m = 0 \\ l + m = 0 \end{array}\right\} \; l = 2k = -m$$

plug in

let's call basic length "a":

$$a^{\frac{3l}{2} + 2m} = m^{k}\, \tilde{e}^{2k}\, \hbar^{-2k}$$

$$\Downarrow$$

$$a^{-k} = m^{k}\, \tilde{e}^{2k}\, \hbar^{-2k} \quad \text{or}$$

$$\boxed{a_B = \frac{\hbar^2}{m\tilde{e}^2} = \frac{4\pi\varepsilon_0\, \hbar}{\cancel{4\pi\varepsilon_0}\, m e^2}}$$

how called? "Bohr radius"

$$a_B = 0.0529177\,\text{nm} \approx 0.5\,\text{Å}$$

- basic time scale

$$\left.\begin{array}{l} k + \frac{l}{2} + m = 0 \\ \frac{3l}{2} + 2m = 0 \end{array}\right\} \; l = 4k, \; m = -3k$$

$$\tau^{-l-m} = \tau^{-k} = m^{k}\, \tilde{e}^{4k}\, \hbar^{-3k} \quad \text{or}$$

$$\tau_B = \frac{\hbar^3}{m\tilde{e}^4} = \frac{(4\pi\varepsilon_0)^2\, \hbar^3}{\cancel{(4\pi\varepsilon_0)^2}\, m e^4} = 2.4 \cdot 10^{-17}\,\text{s}$$

- basic mass scale: m

~~(m =~~

⇒ energy scale:

$$\frac{[M][L]^2}{[T]^2} \rightarrow \frac{m a_B^2}{\tau_B^2} = E_0$$

$$E_0 = \frac{m e^4}{(4\pi\epsilon_0)^2 \hbar^2} = 4.359\,744\,17 \cdot 10^{-18} J$$

⇒ velocity: $\frac{a_B}{\tau_B} = \frac{e^2}{4\pi\epsilon_0 \hbar}$

b) Relativistic atom — what now?

$m, \tilde{e}, \hbar, c$ $\quad \left([c] = \frac{[L]}{[T]} \right)$

either: do over again

or define dimensionless:

$$\alpha = \frac{a_B / \tau_B}{c} = \frac{e^2}{4\pi\epsilon_0 \hbar c} \left(= \frac{1}{137.06} \right)$$

"Fine structure constant"

$E_0 = \alpha^2 m c^2$ (not relativistic!)

c) Important scales:

- length: $\alpha^0 a_B = a_B$: Bohr rad.

  $\alpha^1 a_B = \lambda_c$ (reduced Compton wavelength)

$\alpha^2 a_B \approx r_e$ classical electron radius

- energy:

$\alpha^2 mc^2$ binding energy

$\alpha^4 mc^2$ fine struct. splitting

$\frac{m_e}{m_n} \alpha^4 mc^2$ hyperfine splitting

Use of these basic units ⇒

can set $\hbar = e = m = 4\pi\epsilon_0 = 1$

"atomic units"

Example Schrödinger equation

$$\frac{\hbar^2}{2m} \nabla^2 \psi - \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \psi = E\psi$$

w/ $x = \frac{r}{a_B}$, $\epsilon = \frac{E}{E_0}$:

$$\frac{1}{2} \nabla^2 \psi - \frac{1}{x} \psi = \epsilon \psi$$

ASK ⟹

Values:

m (electron mass)
$= 9.109\,383\,56 \cdot 10^{-31}$ kg ex

e (elementary charge)
$= 1.602\,176\,62 \cdot 10^{-19}$ C ex

$\hbar$ (Planck constant)
$= \frac{1}{2\pi} \cdot 6.626\,070\,15 \cdot 10^{-34}$ J·s ex
$= 1.054\,571\,800 \cdot 10^{-34}$ J·s ex

c (speed of light in vacuum)
$= 299\,792\,458 \ \frac{m}{s}$ ex

ϵ_0 (vacuum permittivity)
$= 8.854\,187\,817 \cdot 10^{-12}$ F/m approx

$a_B = 0.529177$ Å approx

$E_0 = 2.1798723\,6110 \cdot 10^{-18}$ J

$\alpha = 1/137.06$

$\tau_B = 2.4 \cdot 10^{-17}$ s

d) New SI units what are they?

(show kg!) kg, A, m, s, K, mol, cd

Set values for fundamental constants:

kg $h = 6.62607015 \cdot 10^{-34}$ J·s

A $e = 1.602176634 \cdot 10^{-19}$ C

m $c = 299792458 \frac{m}{s}$

^{133}Cs g.s. hyperfine transition:

s $\nu = 9162631770\ s^{-1}$

K $k_B = 1.380649 \cdot 10^{-23}$ J/K

mole $N_A = 6.02214076 \cdot 10^{23}$ /mol

candela K_{cd} (luminous efficacy of 540 THz light) $= 683$ lm/W

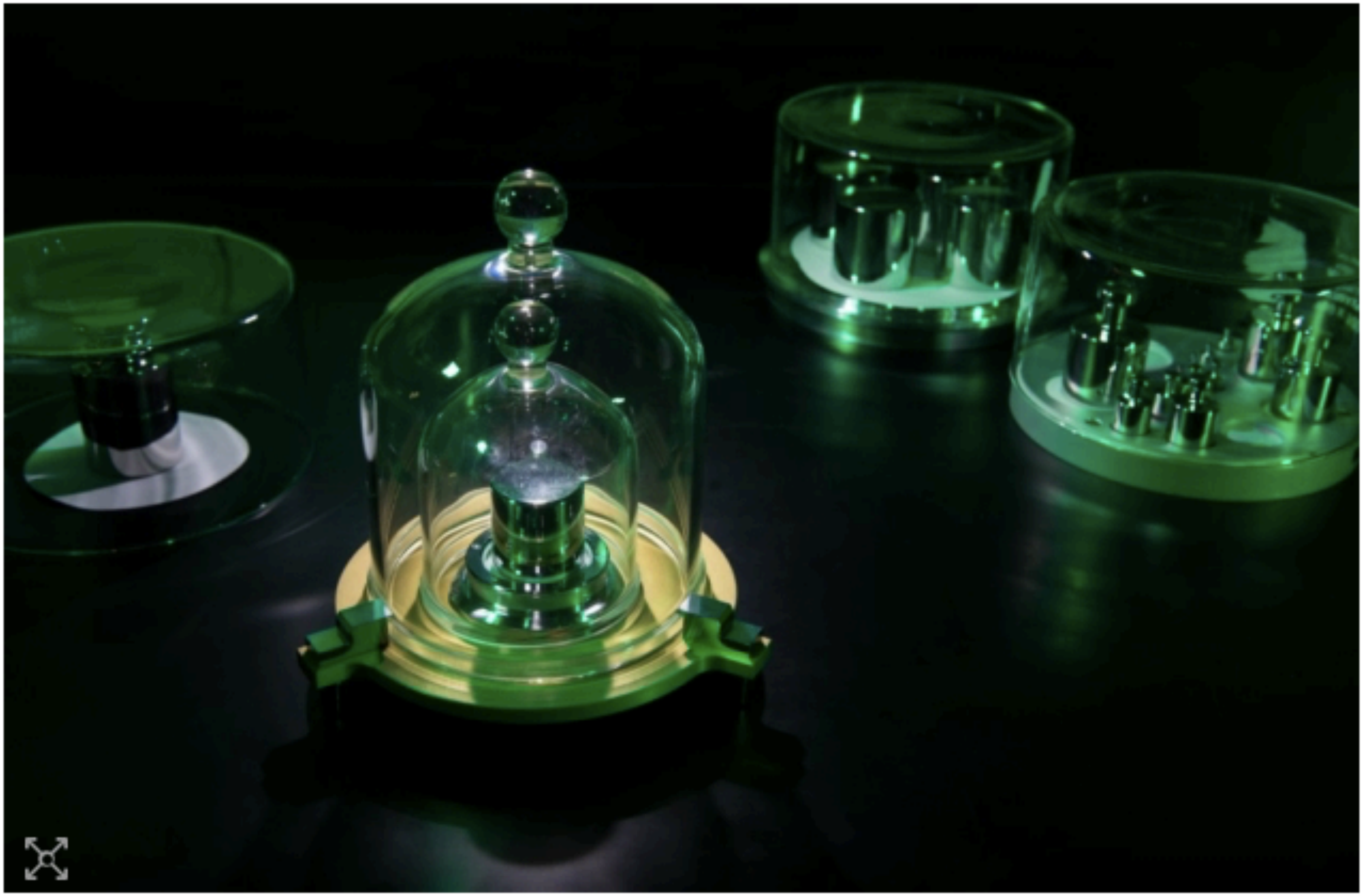
⇒ use to define kg, m, s, A, K, mol, lm

start~~ed~~ May 2019!

"BIPM"

(Bureau International des Poids et Mésures)

Definition of Kilogram?



Weighty matters: a platinum-iridium kilogram belonging to the US National Institute of standards and Technology. (Courtesy: J.L. Lee/NIST)