Lecture 2 - 9/11

Sign up!

Last week: Fundamental quantities (relativistic & not)

answer to question: What fundamental quantities (=D and concepts) do we get in AMO?

(+ review of idea of dimensional analysis)

Today: "transition", "resonance" — how to do spectroscopy (basic principles)

2) Two-level systems & Resonance

a) History of resonances:

1.) measure lines with continuous spectrum (show Balmer)

2) magnetic + electric resonances: (NMR + ESR)

→ much narrower lines

3) advent of laser!

⇒ see Nobel prizes?

Examples (early)

- Lamb shift (suggested Pasternack '37)

$$E(^2S_{1/2}) > E(^2P_{1/2})$$

measured by Lamb in '47

(ESR with ~1% accuracy)

- magnetic moment of electron:

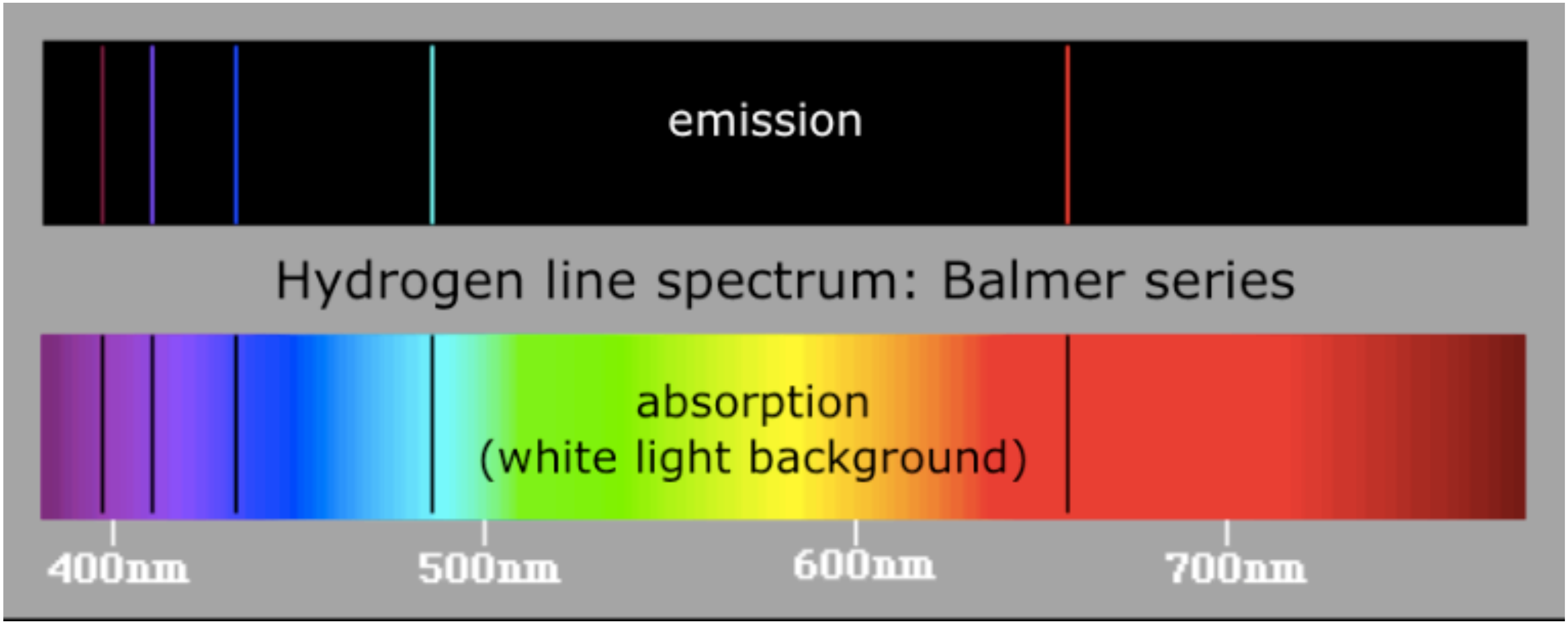
$$\mu_e \overset{?}{=} \mu_B = \frac{e\hbar}{2m}$$

$\left(\frac{\mu_e}{\mu_B} - 1 \approx 1.6 \cdot 10^{-3}\right.$ due to multiplicity of lines etc.)

$\sim$ "$g-2$"

(Kusch '48)

Balmer Series



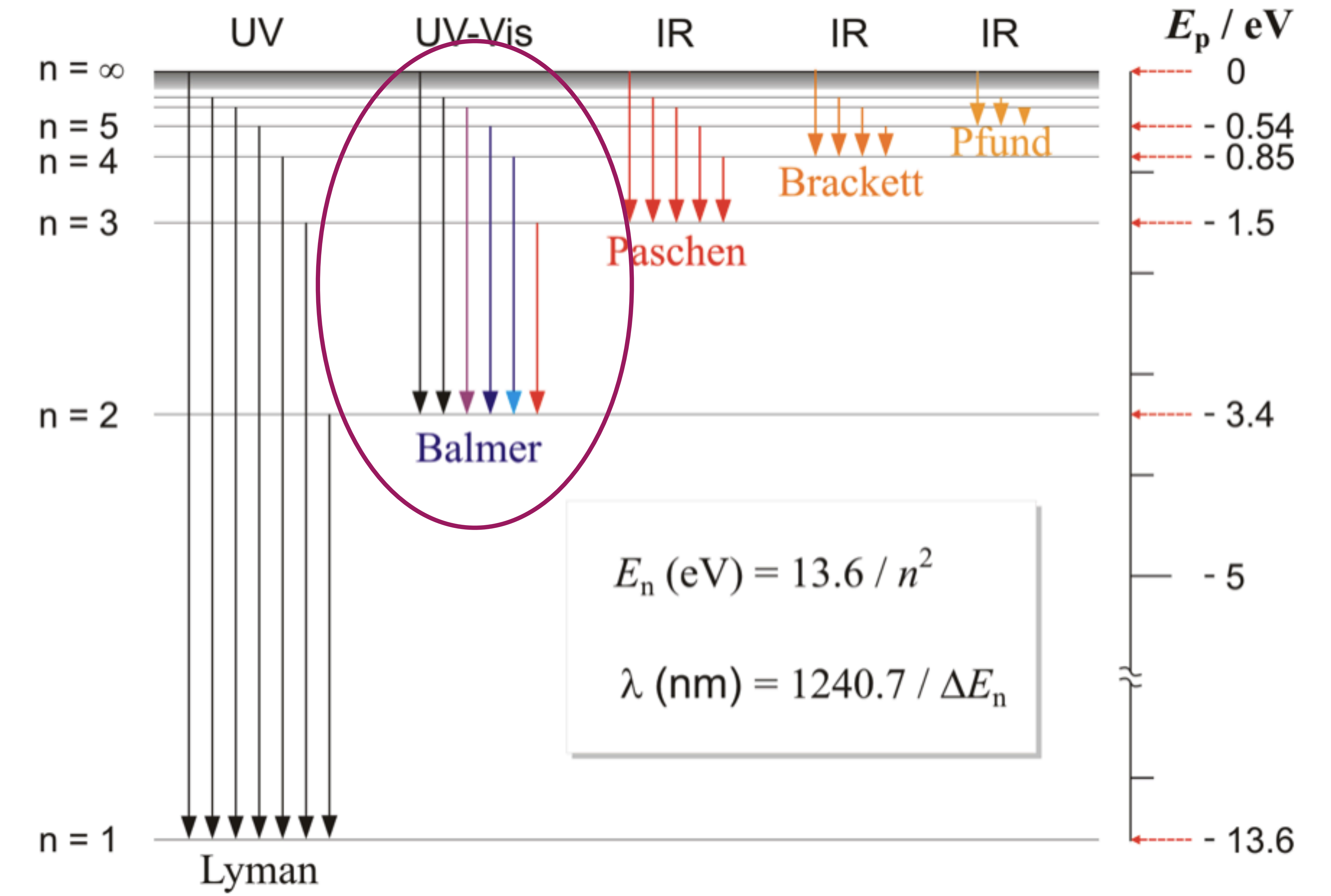
http://www.goiit.com/posts/list/community-shelf-the-bohr-atom-917720.htm

Recent AMO Nobel Prizes

year	Names	topic
1981	Bloembergen, Shawlow	laser spectroscopy
1989	Ramsey, Dehmelt, Paul	hydrogen maser, ion traps
1997	Chu, Cohen-Tannoudji Phillips	cooling and trapping atoms
2001	Cornell, Wieman, Ketterle	Bose-Einstein condensation
2005	Glauber, Hall, Hänsch	quantum theory of light, (frequency comb) spectroscopy
2009	Kao (et al)	optical communication in fibers
2012	Haroche, Wineland	measuring and manipulation of quantum systems
2018	Ashkin, Mourou, Strickland	optical tweezers, high intensity (ultrafast) fields

2022 Aspect, Clauser, Zeilinger – entangled photons, viol. of Bell ineq; pioneering q.i.s.

Hydrogen Spectra



Lamb shift:

What is it? $\rightarrow E(^2S_{1/2}) > E(^2P_{1/2})$

Reason? $\rightarrow$ interaction of e^- w/ el.m. vacuum fluct's

Why is $E(^2S_{1/2})$ larger? $\rightarrow$ $\langle \nabla^2 \frac{1}{r} \rangle$ the larger, the smaller r

El. mgn. moment:

what is it? $g = 2$

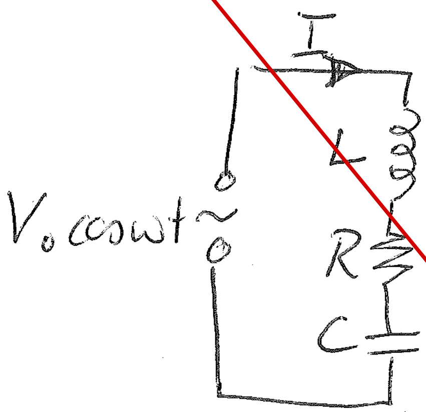
But not quite. Why? $\rightarrow$ presence of degenerate levels

Soon : 10^{-31}

Now: EDM (ACME) $\lesssim 10^{-29}\ e\cdot cm$

b) Classical Resonance; Language

Example of oscillator: LRC-circuit:



voltage change:

$$-L\frac{d}{dt}I$$

$$-IR$$

$$-q/C \qquad \left(I = \frac{d}{dt}q\right)$$

$$\Rightarrow \boxed{\ddot{q} + \underbrace{\gamma}_{\frac{R}{L}}\dot{q} + \underbrace{\omega_0^2}_{\frac{1}{LC}} q = \underbrace{f(t)}_{\frac{V_0}{L}\cos\omega t}}$$

Solve for complex driving/solution:

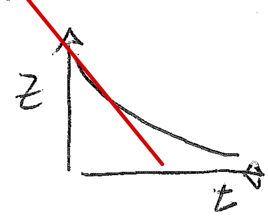
① $\ddot{z} + \gamma\dot{z} + \omega_0^2 z = 0$ (homogeneous)

Ansatz: $z = z_0 e^{\lambda t} \Rightarrow \lambda_\pm = \dfrac{-\gamma \pm \sqrt{\gamma^2 - 4\omega_0^2}}{2}$

3 cases: weakly damped*: $\boxed{\frac{\gamma}{2} < \omega_0}$

critically ": $=$

strongly ": $>$



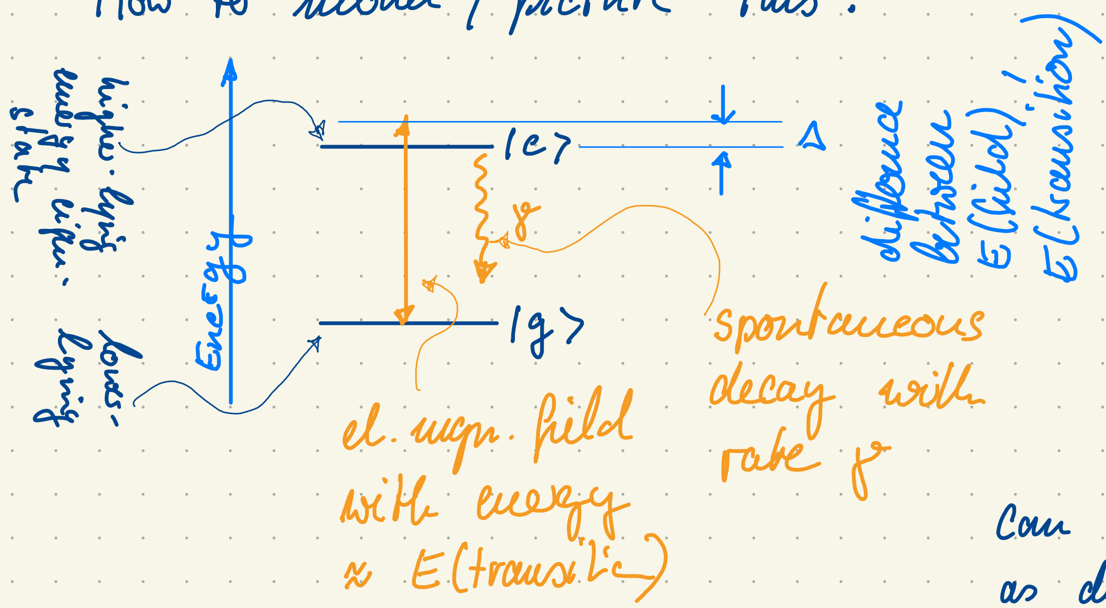
b) Classical resonance; language

What are "resonances"? E(light field) = E(atomic transition)

What is "transition"? e^- moves from one orbit (eigenstate) of atom to another

How does light couple? Atom = ⊕ + ⊖ ⇒ dipole: interacts with el. field / el. magn. field ⇒ energy transfer between light & atom possible

How to model / picture this?



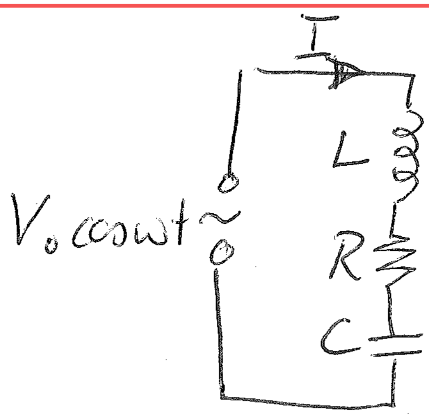
Can be classically modeled as charge oscillating in oscillating field

⇒ like RLC-circuit?

~~Now: EDM (ACME) $\sim 10^{-29}\, e \cdot cm$~~

~~b) Classical Resonance; Language~~

~~Example of oscillator: LRC-circuit:~~



~~voltage~~ current change:

$$-L \frac{d}{dt} I$$

$$-IR$$

$$-q/C \qquad \left(I = \frac{d}{dt} q\right)$$

charge: $\Rightarrow$ $$\boxed{\ddot{q} + \underbrace{\gamma}_{\frac{R}{L}} \dot{q} + \underbrace{\omega_0^2}_{\frac{1}{LC}} q = \underbrace{f(t)}_{\frac{V_0}{L}\cos\omega t}}$$

Solve for complex driving/solution:

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Ansatz: $z = z_0 e^{\lambda t} \Rightarrow \lambda_\pm = \dfrac{-\gamma \pm \sqrt{\gamma^2 - 4\omega_0^2}}{2}$

3 cases: weakly damped*: $\boxed{\frac{\gamma}{2} < \omega_0}$

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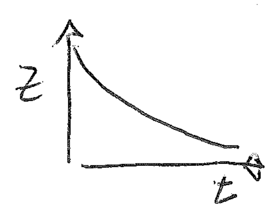


* for atomic case:

$\gamma \ll \omega_0 \approx \frac{1}{\tau_B}$

$\Rightarrow \lambda_\pm = -\frac{\gamma}{\alpha} \pm i\tilde{\omega}_0$

$\tilde{\omega}_0 = \omega_0\sqrt{1-\left(\frac{\gamma}{\alpha\omega_0}\right)^2}$ shifted!

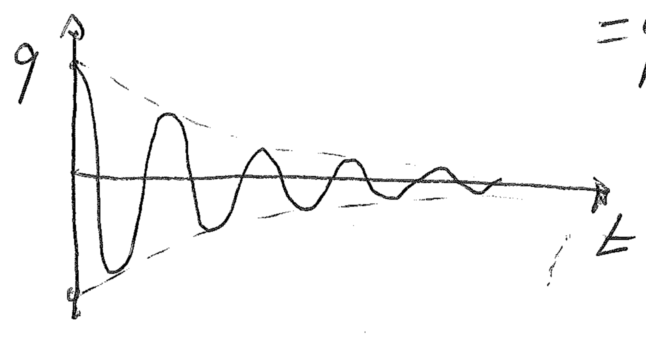
$$\boxed{z(t) = e^{-\frac{\gamma}{\alpha}t}\left(z_+ e^{i\tilde{\omega}_0 t} + z_- e^{-i\tilde{\omega}_0 t}\right)}$$

no drive

damped! oscillation @ $\tilde{\omega}_0 \approx \omega_0$

Real solution for circuit: $q = \mathrm{Re}(z)$

$$= q_0 e^{-\frac{\gamma}{\alpha}t}\cos(\tilde{\omega}_0 t + \varphi)$$



<u>Driven system</u>: $f(t) = f_0 e^{i\omega t}$

$z_{part} = A e^{i\varphi} e^{i\omega t}$:

$$-\omega^2 A e^{i\varphi} e^{i\omega t} + i\omega\gamma A e^{i\varphi} e^{i\omega t} + \omega_0^2 A e^{i\varphi} e^{i\omega t} = f_0 e^{i\omega t}$$

$$\Rightarrow A e^{i\varphi} = \frac{f}{-\omega^2 + i\gamma\omega + \omega_0^2}$$

Amplitude: $\boxed{A^2 = \frac{f^2}{(\omega_0^2-\omega^2)^2 + \omega^2\gamma^2}}$

Phase: $\boxed{\tan\varphi = \frac{\omega\gamma}{\omega^2 - \omega_0^2}}$

$$A^2_{max} = \frac{f^2}{\omega_0^2\gamma^2 - \gamma^4/4}$$

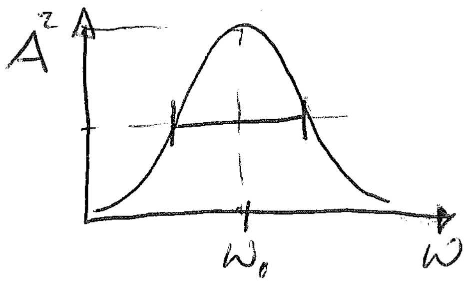
Lorentzian approx:

$$A^2 = \frac{f^2}{(\omega_0-\omega)^2\underbrace{(\omega_0+\omega)^2}_{\approx 2\omega_0} + \underbrace{\omega^2}_{\approx\omega_0^2}\gamma^2}$$

(typically $\gamma^2/\omega_0^2 \sim 10^{-14}$)

$$\boxed{\approx \frac{f^2}{4\omega_0^2}\;\frac{1}{(\omega_0-\omega)^2 + (\gamma/2)^2}}$$

- Full width at half maximum (FWHM)



FWHM = $\boxed{\Delta\omega = \gamma}$

"line width"

$A^2 \propto$ power, energy

Inset:

Full calculation

Driven system: $f(t) = f_0 e^{i\omega t}$

Ansatz for particular solution: $z = A e^{i\varphi} e^{i\omega t}$

$$\Rightarrow -\omega^2 A e^{i\varphi} \cancel{e^{i\omega t}} + i\omega\gamma A e^{i\varphi} \cancel{e^{i\omega t}} + \omega_0^2 A e^{i\varphi} \cancel{e^{i\omega t}} = f_0 \cancel{e^{i\omega t}}$$

$$\Rightarrow A e^{i\varphi} = A\cos\varphi + i A\sin\varphi = \frac{f_0}{\omega_0^2 - \omega^2 + i\omega\gamma}$$

set $\frac{A}{f_0} \in \mathbb{R}$ $\Rightarrow$ then φ describes exactly the phase between the driving field and the atomic response:

$$\left.\begin{aligned} \mathrm{Re}\,\frac{1}{\omega_0^2-\omega^2+i\omega\gamma} &= \frac{\omega_0^2-\omega^2}{(\omega_0^2-\omega^2)^2+\omega^2\gamma^2} \\ \mathrm{Im}\,\frac{1}{\omega_0^2-\omega^2+i\omega\gamma} &= \frac{-\omega\gamma}{(\omega_0^2-\omega^2)^2+\omega^2\gamma^2} \end{aligned}\right\} \quad \begin{aligned} \tan\varphi &= \frac{\sin\varphi}{\cos\varphi} = \frac{\mathrm{Im}}{\mathrm{Re}} \\ &= \frac{\omega\gamma}{\omega^2-\omega_0^2} \approx \frac{\gamma/2}{\Delta} \end{aligned}$$

$\Rightarrow$ $\Delta \to -\infty$: $\tan\varphi \to \pi$

$\Delta \to +\infty$ $\tan\varphi \to 0$

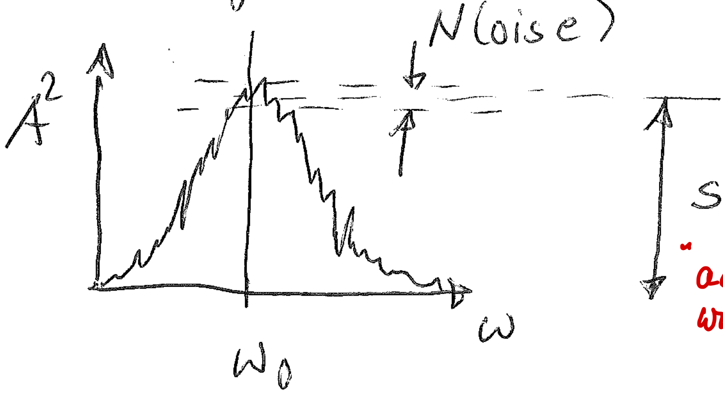
Q: How well can we determine ω_0 experimentally?

A: arbitrarily well (if curve is clean)

Quality factor : $\boxed{Q \equiv \frac{\omega_0}{\Delta\omega}}$

$\tau = \frac{1}{\gamma}$ ~ "decay time"

Usually in experiment:



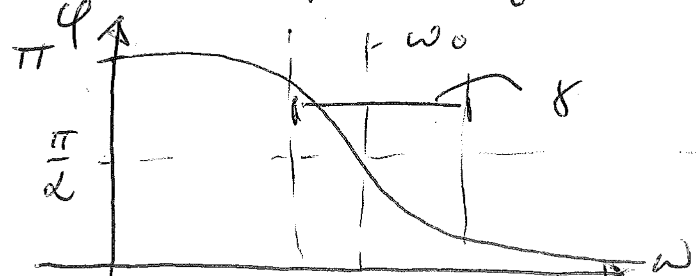
"accuracy" of measurement: width of spectrum. $\Delta\omega$

"Splitting the line": in reality, we can only measure to $\delta\omega$ ($< \Delta\omega$)

$$\delta\omega = \frac{\Delta\omega}{S/N}$$

"signal-to-noise (ratio)" "SNR"

Phase $\tan\varphi \approx \frac{-\gamma/2}{\omega_0 - \omega}$ $= \pm\infty$ @ resonance i.e. $\varphi = \frac{\pi}{2}$



[cf. swing] when do you push, what happens?

Examples:

Earth rotation: $Q \approx 10^7$

Pulsar: 10^{10}

AMO: "useful resonances"
high Q
reproducible
connected to fundamental question
→ change in fundamental constants
e.g. fine struct. c. α : $\boxed{\dot{\alpha}} < 10^{-15}$/year
→ sometimes surprises: e.g. Zeeman (mid 1896)
→ "anomalous" Zeeman (Preston, 1898)
→ Lamb shift

- typical Q for visible due to Doppler effect @ RT: $Q \approx 10^6$

- e^- in Penning trap: $10^6 - 10^7$

- H 1s – 2p line $\approx 2 \cdot 10^7$

- He fine structure $\sim 10^8 - 10^9$

Remarks:

- $\omega = 2\pi f$ f: frequency
  ⇒ ω measured in "$\frac{rad}{s}$" or "$1/s$"
  f measured in Hz
  or even "$\omega = 2\pi \cdot 10$ MHz"
  → $\omega = 62.8 \cdot 10^6 \, 1/s$

- γ is given as "$1/s$", more often $1/\gamma \, (=\tau)$ as "s"

$\Delta\omega \cdot \tau \equiv 1$

But what if measuring time

$\Delta t < \infty$?

$\Rightarrow \Delta\omega \cdot \Delta t \geq 1/2$ (due to Fourier-trafo or uncertainty)