Lecture 4 – 9/18

Finish simple TLS calc:

- → interaction pic eqs of motion
- → particular light field + dipole interaction Ham.
- → specific cases of weak + strong excitation
- → Bloch sphere
- → light shifts
- → dressed states
- → Adiabaticity

Note: These concepts are of much wider impact than TLS!

(2) mit (1)

$$\langle n| i\hbar \sum_m (\dot{c}_m - i\omega_m c_m) e^{-i\omega_m t} |m\rangle =$$

$$\langle n| \sum_m c_m \hbar\omega_m e^{-i\omega_m t} |m\rangle + \langle n| \sum_m V c_m e^{-i\omega_m t} |m\rangle$$

$$i\hbar \dot{c}_n = \sum_m e^{-i(\omega_m - \omega_n)t} \underset{\langle n|V|m\rangle}{V_{nm}}$$

TLS:

$$\boxed{\begin{aligned} i\hbar \dot{c}_e &= c_g e^{i\omega_0 t} V_{eg} \\ i\hbar \dot{c}_g &= c_e e^{-i\omega_0 t} V_{ge} \end{aligned}} \quad (*)$$

Assume rotating $V_{eg}(t) \equiv \frac{\hbar}{2} \Omega_R e^{-i\omega t} = V_{ge}(t)^*$

$$\left.\begin{aligned} i\cancel{\hbar} \dot{c}_e &= \frac{\cancel{\hbar}}{2} \Omega_R e^{-i(\omega - \omega_0)t} c_g \\ i\cancel{\hbar} \dot{c}_g &= \frac{\cancel{\hbar}}{2} \Omega_R^* e^{i(\omega - \omega_0)t} c_e \end{aligned}\right\} \omega - \omega_0 \equiv \delta \text{ "detuning"}$$

usually $\delta \ll \omega, \omega_0$

Get rid of time dependence (→ interaction pic)

$$\tilde{c}_e = e^{-i\omega_e t} c_e \quad \text{(w/ arbitrary } \omega_e\text{)}$$

$$\tilde{c}_g = e^{-i\omega_g t} c_g$$

$$\Rightarrow \dot{c}_e = (\dot{\tilde{c}}_e + i\omega_e \tilde{c}_e) e^{i\omega_e t}; \quad \dot{c}_g = (\dot{\tilde{c}}_g + i\omega_g t) e^{i\omega_g t}$$

$$\Downarrow$$

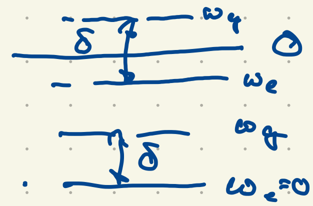
$$i(\dot{\tilde{c}}_e + i\omega_e \tilde{c}_e) = \frac{\Omega_R}{2} \tilde{c}_g e^{-i(\delta - \omega_g + \omega_e)t}$$

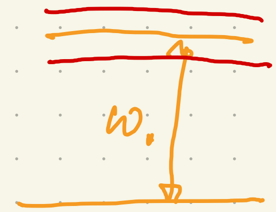
$$(\dot{\tilde{c}}_g + i\omega_g \tilde{c}_g) = \frac{\Omega_R}{2} \tilde{c}_e e^{i(\delta - \omega_g + \omega_e)t}$$

choose $\boxed{\delta \equiv \omega_g - \omega_e}$

Time dependence gone ☺

choose $\omega_{e/g} = \mp \delta/2$





or $\omega_e = 0, \; \omega_g = \delta$

or . . . – – .

$$\boxed{\begin{aligned}\dot{\tilde c}_e &= -i\omega_e \tilde c_e - i\frac{\Omega_R}{2}\tilde c_g \\ \dot{\tilde c}_g &= -i\omega_g \tilde c_g - i\frac{\Omega_R^*}{2}\tilde c_e\end{aligned}}$$

$\tilde c_e, \tilde c_g$, eq. system in "rotating frame"

Hamiltonian:

$$\tilde H = \hbar \begin{pmatrix}\omega_e & \frac{\Omega_R}{2}\\ \frac{\Omega_R^*}{2} & \omega_g\end{pmatrix}$$

(ii) Typical form for V

$$V = -\vec p\cdot\vec E \qquad (**)$$

dipole moment — electric field (laser)

$$\vec E = \vec E^+ e^{-i\nu t} + \vec E^- e^{i\nu t} \qquad \text{(real!)}$$

$$\vec E^{+*} = \vec E^-$$

$$\vec p = e\hat{\vec r} \; : \langle e|\hat{\vec r}|e\rangle = \langle g|\hat{\vec r}|g\rangle = 0 \quad \text{symm!}$$

1D: $\int_{-\infty}^{\infty} dr\, |\psi_e(r)|^2\, r = 0$

odd!

$$\hat p = e\hat r = e\,\mathbb{1}\,\hat r\,\mathbb{1} = e\Big(\sum_i |i\rangle\langle i|\Big)\hat r\Big(\sum_j |j\rangle\langle j|\Big) =$$

$$= e\sum_{i,j} e\, r_{ij}\, |i\rangle\langle j|$$

imp

$\wp \equiv e\, r_{eg}$ "dipole matrix element" coupling constant

$$\overset{TLS}{=} e\, r_{eg}\, |e\rangle\langle g| + h.c.$$

plug in (**)

$$V_{eg} = -\wp \left(E^+ e^{-i\nu t} + E^- e^{i\nu t}\right)$$

$$\Rightarrow i\hbar \dot{c}_e = c_g \left(-\wp \left(E^+ e^{-i(\nu-\omega_0)t} + E^- \cancel{e^{i(\nu+\omega_0)t}}\right)\right)$$

"counter rotating"

for $|\nu-\omega_0| \ll |\nu+\omega_0|$ averages out

fast rotating neglected:

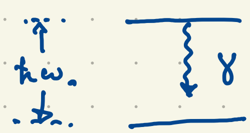
$\boxed{\text{"rotating wave approximation"}}$ RWA

$$\boxed{V_{eg}(t) \approx -\wp E^+ e^{-i\nu t}} \Rightarrow$$

$$\boxed{\frac{\Omega_R}{2} \equiv \frac{\wp E^+}{\hbar}}$$ "Rabi frequency"

- similar $V = -\vec{\mu}\cdot\vec{B}$ rotating $B(t)$ (ex: spin in rotating magn field)

f) Weak near-resonant excitation

allow decay! 

phenomenologically: $|e\rangle \to e^{-\frac{\gamma}{2}t}|e\rangle$

ad hoc "patch" into S.Eq.: $e^{\pm i\omega_0 t} \to e^{\pm i\omega_0 t - \frac{\gamma}{2}t}$

$c_g(t) \approx c_g(0) = 1$, $c_e(t) \ll 1$
perturbation

$\Rightarrow$

plug into eqs. of motion (*)

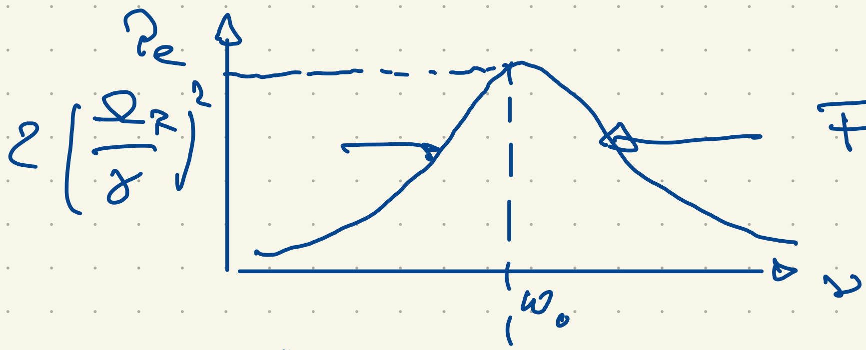
$$c_e(t) = -i\int_0^t dt' \frac{\Omega_R}{2} e^{-i\delta t' - \frac{\gamma}{2}t'} =$$

$$= -i\frac{\Omega_R}{2}\frac{1}{-i\delta - \gamma/2}\left(e^{-i\delta t - \frac{\gamma}{2}t} - 1\right)$$

$$\overset{t\to\infty}{=} -\frac{\Omega_R}{2}\frac{1}{\delta - i\gamma/2}$$

$$\boxed{P_e = |c_e(t\to\infty)|^2 = \frac{|\Omega_R|^2}{2}\frac{1}{\delta^2 + (\gamma/2)^2}}$$

"transition probability" (Lorentzian)



FWHM $= \gamma$

⇒ "Weak" means $|\Omega_R| \ll \gamma$

Hamiltonian formulation

$$\tilde{H} = \hbar\begin{pmatrix} -\delta - i\frac{\gamma}{2} & \Omega_R/2 \\ \frac{\Omega_R^*}{2} & 0 \end{pmatrix}$$

"non-hermitian Hamiltonian"

"exact solution with this Hamiltonian:

$$\tilde{c}_e = \frac{-i\Omega_R}{2\sqrt{(\gamma_2 + i\delta)^2 - \Omega_R^2}} \cdot e^{\frac{1}{2}\left(-\frac{\gamma}{2} + \sqrt{(\gamma_2 + i\delta)^2 - |\Omega_R|^2}\right)t} \cdot \left(1 - e^{-\sqrt{(\frac{\gamma}{2} + i\delta)^2 - |\Omega_R|^2}}\right)$$

(for $|\Omega_R| \ll \gamma$: same as before!)

works also for "non-weak" excitation, but only if decay is outside the system!

g) Strong excitation

$\gamma \ll |\Omega_R|$ neglect decay (i.e. fast enough unitary evolution)

$$H = \begin{pmatrix} -\delta & \frac{\Omega_R}{2} \\ \frac{\Omega_R^*}{2} & 0 \end{pmatrix}$$

on resonance : $\delta = 0$

$$\dot{\tilde{c}}_e = -i \frac{\Omega_R}{2} \dot{\tilde{c}}_g = -\frac{|\Omega_R|^2}{4} \tilde{c}_e$$

$$\dot{\tilde{c}}_g = -i \frac{\Omega_R^*}{2} \tilde{c}_e$$

harmonic oscillator