As usual, if you use ChatGPT (or any other generative AI), you need to tell us on your homework!

Physics 285a Problem Set 2

posted September 20, 2023, due September 26, 2023

Problem 1. Off-resonant Rabi oscillations: During Wednesday's class, we have a brief look at Rabi oscillations on resonance. What happens if we drive a bit off-resonance? What happens to the population transfer? What is the Rabi oscillation frequency compared to the on-resonance case? Can you try to give a somewhat intuitive explanation for this? *Feel free to compare your ideas to last year's lecture notes and/or ChatGPT. If you do so, please use your own text and let us know the difference in explanation for both sources.*

Problem 2. Rotating Wave Approximation: The rotating wave approximation is frequently employed in AMO physics. When transitions between two states are involved, an oscillating driving term in the Hamiltonian that causes transitions between these two states is approximated as

$$V(t) = \frac{1}{2}\hbar\Omega_R \left[\frac{\sigma_x + i\sigma_y}{2} e^{-i\phi(t)} + \frac{\sigma_x - i\sigma_y}{2} e^{i\phi(t)} \right]. \tag{1}$$

The σ_i are Pauli matrices that operate in the space of the two states. The strength of the driving term is Ω_R, the Rabi frequency.

(a) **Format** Describe briefly what the various terms are in this form (feel free to check with lecture notes, text books, ChatGPT...) What condition do we have for the phase $\phi(t)$ such that the system can be called "near resonant," i.e., a rotating wave approximation makes sense for this case? Try to write the general formulation of the equations of motion in the Schrödinger and then in the interaction picture, similar as we did in class. Make the rotating wave approximation in this general case and write the Hamiltonian with the RWA in matrix form. (This is also the form that you can then use for parts (b) – (d).)

(b) **Rabi Frequency for Hydrogen 1s-2p:** A linearly polarized plane wave (e.g., pointing along $\hat{z}$) from a laser illuminates a hydrogen atom in its ground state. Neglect hyperfine and fine structure. What is the Rabi frequency for the 1s-2p transition as a function of the intensity of the laser (power per unit area)? Clearly identify the terms that have been neglected.

(c) **Rabi Frequency for an Electron Suspended in a Uniform Magnetic Field:** Consider an electron suspended in a strong and uniform magnetic field $B_z = B$ with a second oscillating magnetic field that is perpendicular to the first, with $b_x = b\cos(\omega t)$. Write down the interaction term between the electron and the total magnetic field. As a next step make the rotation wave approximation. Specify the Rabi frequency and clearly identify the terms that have been neglected.

(d) **$\mathbf{NH_3}$**: Deduce real fields that can drive the nitrogen atom in an NH_3 molecule between its two stable states in a large static electric field. To do so, first consider the effect of the static electric field E_0 with no additional driving. Diagonalize the resulting Hamiltonian to determine the eigenstates in the presence of E_0. Adding an oscillating field, make the rotating wave approximation. Identify the parameters that will characterize the transition and explicitly identify any terms that are neglected. Discuss the importance of the static electric field.