Lecture 7 — 9/27

Homework: not more than 10-15 hrs max (10 ave)

How to shorten: collaborate

ChatGPT

ask for help

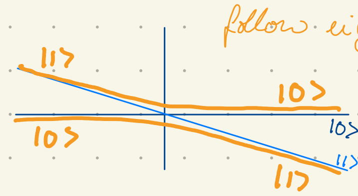
⋮

Finish Landau-Zener crossing

Open systems

Landau - Zener crossing ← time - dependent detuning

follow eigenstate



$$i\dot{c}_e = -\frac{\Omega_R}{2} e^{-i\delta(t)t} c_g$$

$$i\dot{c}_g = -\frac{\Omega_R^*}{2} e^{i\delta(t)t} c_e$$

example : $\delta(t) = \alpha$ (linear sweeps)

$$0 = \ddot{c}_{e/g} \pm 2i\alpha t\, \dot{c}_{e/g} + \frac{|\Omega_R|^2}{4} c_{e/g}$$

$$\tau \equiv \frac{|\Omega_R|}{2} t$$

$$\boxed{0 = c''_{e/g} \mp 2i\frac{\tau}{\Gamma} c'_{e/g} + c_{e/g}}$$

only scale is Γ

$$\boxed{\Gamma \equiv \frac{|\Omega_R|^2}{4\alpha}}$$

"Landau - Zener parameter"

(exact : Zener : Proc. Roy. Soc. London A, 137, 696 (1932).
approx : Vutha, arXiv : 1001.3322

Perturbation sol. : Γ small ; $c_g(-\infty) = 1$

i.e. change fast

$$\left(\frac{\Omega_R}{|\Omega_R|} = e^{i\varphi} \right)$$

$$c'_{e/g} = i\, c_{g/e}\, e^{\mp i(\varphi - \frac{4\alpha}{|\Omega_R|^2}\tau^2)}$$

$$= i\, c_{g/e}\, e^{-i(\varphi - \frac{\tau^2}{\Gamma})}$$

$$\Rightarrow c_e(\infty) = c_e(-\infty) + \int_{-\infty}^{\infty} d\tau \left(-i \underbrace{c_g(\tau)}_{\approx 1} e^{-i(\varphi - \frac{\tau^2}{\Gamma})} \right)$$

narrow Gaussian

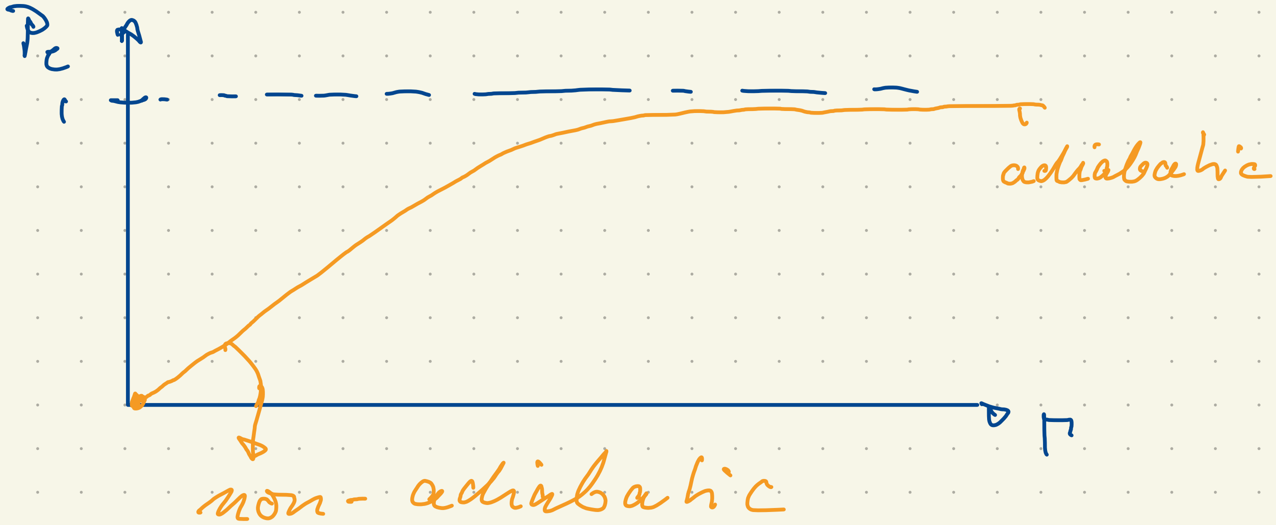
$$c_e(\infty) \approx -i e^{-i\varphi_0} \sqrt{i\pi\Gamma}$$

$$P_e(\infty) = |c_e(\infty)|^2 \approx \pi\Gamma$$

$$\Rightarrow P_g(\infty) \approx 1 - \pi\Gamma$$

not Γ small

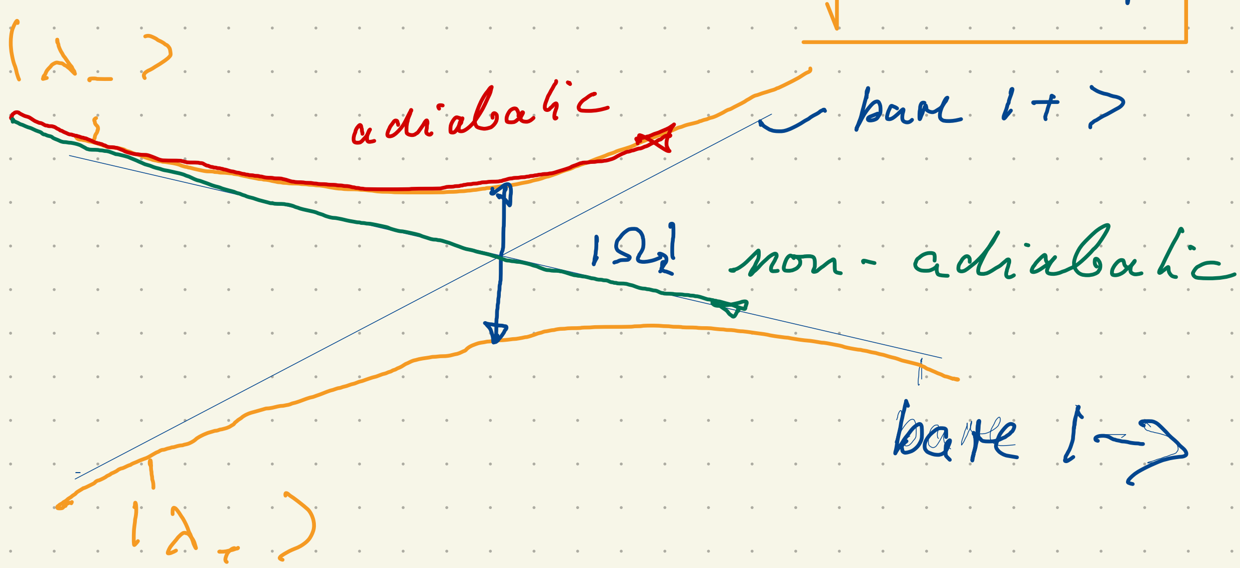
(educated guess : $P_g \approx e^{-\pi\Gamma}$; $\boxed{P_e \approx 1 - e^{-\pi\Gamma}}$)



fast sweep (Γ small) $\Rightarrow$ no transition

slow sweep (Γ large) $\Rightarrow$ transition

$$|\dot{\delta}| = \underbrace{|\alpha|}_{= \frac{|\Omega_R|^2}{4\Gamma}} \ll |\Omega_R|^2 \quad \Rightarrow \quad \boxed{\Gamma \gg 1}$$



NB: "bare states" = eigenstates of H_0

"dressed states" = eigenstates of $H = H_0 + V$

3.) Open Systems & Density operator

In principle, all problems can be solved using wave functions, but this can be cumbersome ...

$\Rightarrow$ find treatment with limited "system," where "environment" can be traced out without sacrificing accuracy.

New description of state via "ρ":

"density operator" or "density matrix"

<u>a) Properties</u> for system in state $|\psi\rangle$:

$$\boxed{\rho = |\psi\rangle\langle\psi|}$$

Thus, where $|\psi\rangle$ is a "state," ρ is an "operator." If $|\psi\rangle$ is represented as a vector, ρ is represented as a matrix.

<u>EOM:</u> Start from Schr. Eq.: $(i\hbar|\dot{\psi}\rangle = H|\psi\rangle)$

$$\dot{\rho} = |\dot{\psi}\rangle\langle\psi| + |\psi\rangle\langle\dot{\psi}| =$$

$$= \frac{1}{i\hbar}\left(H|\psi\rangle\langle\psi| - |\psi\rangle\langle\psi|H\right) =$$

$$= \boxed{\frac{1}{i\hbar}[H, \rho] = \dot{\rho}}$$

Von Neumann Eq.

($\hat{=}$ Schr. Eq.)

ρ must satisfy:

1.) $\rho = \rho^{+}$

2.) $\mathrm{Tr}\,\rho \equiv \sum_n \langle n|\rho|n\rangle = 1$ if $\{|n\rangle\}$ is a basis

3.) $\mathrm{Tr}\,\rho^2 = \mathrm{Tr}\,\rho = 1$ (and $\rho^2 = \rho$: "projector")

In this case, for example:

$$|\psi\rangle = \alpha|e\rangle + \beta|g\rangle \quad \Rightarrow$$

$$\rho = \rho_{ee}|e\rangle\langle e| + \rho_{eg}|e\rangle\langle g| + \rho_{ge}|g\rangle\langle e| + \rho_{gg}|g\rangle\langle g|$$

where $\rho_{ij} \equiv \langle i|\rho|j\rangle$ $\Rightarrow$

$$\rho_{ee} = |\alpha|^2 = P_e \;;\quad \rho_{eg} = \alpha\beta^*$$

$$\rho_{gg} = |\beta|^2 = P_g \;;\quad \rho_{ge} = \alpha^*\beta$$

Note: here $|e\rangle\langle g|$ connects $|e\rangle$ and $|g\rangle$ *coherently*, i.e., with *fixed phase*. In general, phase relation is expected to get lost, e.g. in thermal equilibrium:

$$\rho_{eg}^{\text{thermal}} = \rho_{ge}^{\text{thermal}} = 0 \;;\quad \rho_{ee}/\rho_{gg} = e^{-\frac{\Delta E}{k_B T}} \quad \text{(Boltzmann)}$$

$$\Rightarrow \rho^{\text{thermal}} \propto \begin{pmatrix} e^{-\frac{\Delta E}{2kT}} & 0 \\ 0 & e^{\frac{\Delta E}{2kT}} \end{pmatrix};$$

Thus, ρ^{thermal} *cannot* be written in the form $\rho = |\psi\rangle\langle\psi|$.

As a more general definition:

$$\boxed{\rho = \sum_\alpha p_\alpha |\psi_\alpha\rangle\langle\psi_\alpha|},$$

with p_α the probability to be in state $|\psi_\alpha\rangle$. If there is more than one $p_\alpha > 0$, the system is said to be in a "mixed state" and ρ cannot be written as $|\psi\rangle\langle\psi|$.

Thus, the set of properties need to be revised:

$$\boxed{\begin{array}{lll} 1) & \rho = \rho^{+} & \text{(as above)} \\ 2) & \mathrm{Tr}\,\rho = 1 & (\quad " \quad) \\ 3) & \mathrm{Tr}\,\rho^2 \leq 1 & (\mathrm{Tr}\,\rho^2 = 1: \text{ pure state} \\ & & \;\mathrm{Tr}\,\rho^2 < 1: \text{ mixed state}) \end{array}}$$

<u>Expectation values</u> of observable $\hat{O}$:

$$\langle \hat{O} \rangle \equiv \sum_\alpha p_\alpha \langle \psi_\alpha | \hat{O} \underbrace{\;}_{\mathbb{1}} | \psi_\alpha \rangle =$$

$\{|n\rangle\}$: basis

$$= \sum_{n,\alpha} p_\alpha \langle \psi_\alpha | \hat{O} | n \rangle \langle n | \psi_\alpha \rangle =$$

$$= \sum_n \langle n | \underbrace{\sum_\alpha |\psi_\alpha\rangle\langle\psi_\alpha| p_\alpha}_{\rho} \hat{O} | n \rangle =$$

$$= \boxed{\mathrm{Tr}\,\rho\,\hat{O} = \overline{O}}$$

and

$$\boxed{\rho = \sum_{n,m} |n\rangle \underbrace{\langle n| \rho |m\rangle}_{\rho_{nm}} \langle m| = \sum_{n,m} |n\rangle\langle m| \rho_{nm}}$$

$(\{|n\rangle\}, \{|m\rangle\} \mathrel{\hat{=}} \text{basis})$

<u>Physical interpretation:</u>

1) $n = m$: ρ_{nm} is probability
2) $n \neq m$: "coherence" between $|n\rangle$ and $|m\rangle$
(e.g. dipole operator $|e\rangle\langle g|$)

b) System + Environment

total system (universe) $\to |\psi\rangle$

= "system" + "reservoir" / "environment"

Goal: keep as much of environment as is relevant for system evolution.

basis: $\{|n\rangle\} \otimes \{|e\rangle\}$

$$|\psi\rangle = \sum_{\{n,e\}} |n,e\rangle\langle n,e|\psi\rangle$$

Assume $\hat{S}$ acts only on system:

$$\langle\hat{S}\rangle = \langle\psi|\hat{S}|\psi\rangle =$$

$$= \sum_{\{n,e\}}\sum_{\{n',e'\}} \langle\psi|n,e\rangle \underbrace{\langle n,e|\hat{S}|n',e'\rangle}_{\langle n|\hat{S}|n'\rangle\,\delta_{ee'}} \langle n',e'|\psi\rangle$$

$$= \sum_{\{n\}\{n'\}} \langle n|\hat{S}|n'\rangle\langle n'| \underbrace{\sum_{\{e\}}\langle e|\psi\rangle\langle\psi|e\rangle}_{\mathrm{Tr}_{env}\,\rho_{total} \equiv \rho_{sys}} |n\rangle$$

$\mathbb{1}$

$$= \boxed{\mathrm{Tr}_{sys}\,\rho_{sys}\,\hat{S} = \langle\hat{S}\rangle}$$

with $\rho_{sys} \equiv \mathrm{Tr}_{environ}\,|\psi\rangle\langle\psi|$

ρ_{sys} has all properties (1,2,3) of a density operator.

- Dynamics:

$$i\hbar\dot{\rho} = \sum_\alpha p_\alpha\left(i\hbar|\dot{\psi}\rangle\langle\psi| + i\hbar|\psi\rangle\langle\dot{\psi}|\right) =$$

$$= \sum_\alpha p_\alpha\left(H|\psi\rangle\langle\psi| - |\psi\rangle\langle\psi|H\right) = [H,\rho]$$

system + environment:

$$\dot{\rho}_{sys} = \frac{1}{i\hbar}\left[H_{sys} + \cancel{H_{envir.}} + H_{sys-env}, \rho_{sys}\right]$$

$\cancel{H_{envir.}} \to 0$: commutes w/ ρ_{sys}

<u>Interaction picture</u>

$$U_0 = e^{-\frac{i}{\hbar}H_0 t} : \tilde{\rho} = U_0^{+}\rho U_0, \quad \tilde{H}_{int} = U_0^{+} H_{int} U_0$$

for any $H = H_0 + H_{int}$

here: $H_0 \hat{=} H_{sys} + H_{env}$; $H_{int} \hat{=} H_{sys-env.}$

$$\Rightarrow U_0 = \exp\left[-\frac{i}{\hbar}(H_{sys} + H_{env})\right]$$

$$\dot{\tilde{\rho}}_{sys} = \frac{1}{i\hbar}\left[\tilde{H}_{sys-env}, \tilde{\rho}_{sys}\right]$$

c) Kraus operators & Lindblad Eq.

$\rho_{s(ys)} = Tr_{E(nv)}\, \rho_{total}$; assume: $\rho_{total} = \rho_s \otimes |e\rangle\langle e|$

at some time t_0

<u>Properties:</u>

- M_k is system operator
- $\sum_k M_k^{+} M_k = \sum_k \langle e| U_{SE}^{+} |e_k\rangle\langle e_k| U_{SE} |e\rangle =$

$$= \langle e| U_{SE}^{+} U_{SE} |e\rangle = \mathbb{1}_S$$