3.13 Transit-time broadening

A beam of atoms moving with velocity $\vec{v} = v\hat{x}$ crosses a laser beam propagating along $\hat{y}$. The cw narrow-band laser beam has frequency ω_L, its z-dimensions are greater than those of the atomic beam, and its intensity is $I(x,z) = I_0$ for $-w < x < w$ and zero elsewhere. Assume that the laser light is of sufficiently low intensity so that all saturation effects can be ignored. Also assume that the density of atoms in the beam is low enough so that the atomic beam may be treated as an optically thin medium.

(a) Estimate the broadening of the absorption line due to the finite time of interaction between atoms and the light (transit-time broadening).

(b) Suppose the laser is tuned to a transition between the atomic ground state and an excited state (separated in energy by $\hbar\omega_0$) with radiative lifetime τ. For $v = 5 \times 10^4$ cm/s and diameter $2w = 1$ mm, estimate for which values of τ transit-time broadening effect will dominate the line width.

(c) Using a classical and/or quantum mechanical picture, explain the additional lobes on the spectral profile of a transit-broadened line (see Fig. 3.20). Assume that the lifetime of the excited state τ greatly exceeds the transit time $\sim 2w/v$.

(d) What is the spectrum of a transit-broadened line if we instead assume a Gaussian spatial profile for the laser beam, i.e., $I(x,z) = I_0 e^{-2x^2/w^2}$? (The factor of 2 appears in the exponent because the beam radius is conventionally defined to correspond to the $1/e$ level for the electric field amplitude.)

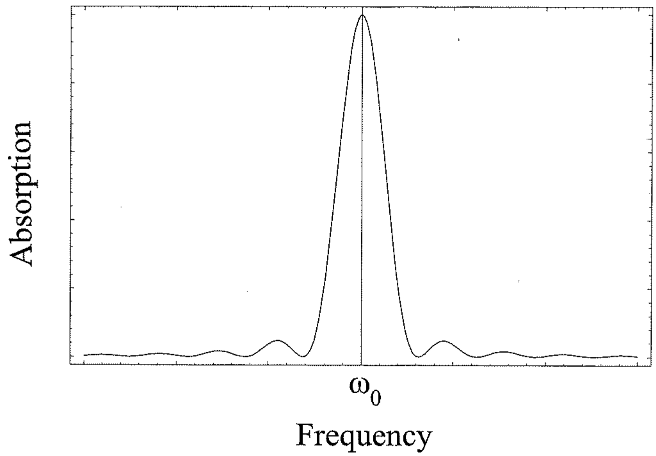

FIG. 3.20 Absorption spectrum for atomic transition where the lineshape is due to transit-time broadening.

Solution

(a) An atom traversing the laser beam "sees" a pulse of radiation with a duration $2w/v$. This means that the effective radiation spectrum is broadened according to the uncertainty condition

$$\Delta\nu\Delta t \sim \frac{1}{2\pi} \,. \tag{3.238}$$

Therefore,

$$\boxed{\Delta\nu_{\text{transit}} \sim \frac{v}{4\pi w} \,.} \tag{3.239}$$

(b) For $v = 5 \times 10^4$ cm/s and $2w = 1$ mm, this corresponds to

$$\Delta\nu_{\text{transit}} \sim 0.1 \text{ MHz} \,. \tag{3.240}$$

The radiative width is

$$\Delta\nu_{\text{radiative}} = \frac{1}{2\pi\tau} \,. \tag{3.241}$$

Therefore, the transit broadening dominates when

$$\boxed{\tau \gg 2\ \mu\text{s} \,.} \tag{3.242}$$

(c) Suppose that the laser light phase is such that its electric field at the atom is $\mathcal{E}(t) = \mathcal{E}_0 \cos(\omega_L t)$. The intensity spectrum of this radiation can be found by taking the Fourier component:[12]

$$\mathcal{E}(\omega) = \int_{t=-w/v}^{t=w/v} \mathcal{E}_0 \cos(\omega_L t') e^{-i\omega t'} dt' \,, \tag{3.243}$$

and evaluating the quantity $I(\omega) \propto \mathcal{E}(\omega)\mathcal{E}(\omega)^*$. One finds, after some math, that

$$\boxed{I(\omega) \propto \frac{\sin^2\left[(\omega - \omega_L)w/v\right]}{(\omega - \omega_L)^2} \,.} \tag{3.244}$$

[12] The Fourier transform or the *spectral distribution* of an arbitrary time-dependent function $F(t)$ is defined as

$$F(\omega) = \int_{-\infty}^{\infty} F(t) e^{-i\omega t} dt.$$

The inverse transformation is

$$F(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega.$$

This function is centered around the laser frequency ω_L. If one scans the laser light frequency through the atomic resonance, the absorption spectrum [resulting from the spectral intensity distribution (3.244)] is that shown in Fig. 3.20 (in the limit where $\tau \gg w/v$).

The lobes on the spectral profile are analogous to those that appear in the diffraction of light from a thin slit. In the case of a thin slit, the properties of a monochromatic field are modified by limiting the extent of the field in space, while in transit-time broadening, the properties of the field are modified by limiting the extent of the field in time.

(d) In this case, the time-dependent electric field seen by atoms passing through the laser beam is given by

$$\mathcal{E}(t) = \mathcal{E}_0 \cos{(\omega_L t)} e^{-v^2 t^2/w^2}\,, \tag{3.245}$$

where we have transformed the spatial dependence of the laser beam intensity into the time-dependence of the electric field by setting $\mathcal{E}(t)$ equal to the square root of $I(x = vt, z)$. As in part (c), we take the Fourier transform of $\mathcal{E}(t)$:

$$\mathcal{E}(\omega) = \int_{-\infty}^{\infty} \mathcal{E}_0 \cos{(\omega_L t')} e^{-v^2 t'^2/w^2} e^{-i\omega t'} dt'\,, \tag{3.246}$$

and for $I(\omega) \propto \mathcal{E}(\omega)\mathcal{E}(\omega)^*$ we obtain

$$I(\omega) \propto e^{-w^2(\omega-\omega_L)^2/(2v^2)}\,, \tag{3.247}$$

where we have ignored far-off resonant terms involving factors of $\exp[-w^2(\omega + \omega_L)^2/(2v^2)]$. Thus the spectral profile in this case is a Gaussian. Note that the intensity drops to the $1/e$ point at $|\omega - \omega_L| = \sqrt{2}(v/w)$, whereas the first zeros of the profile shown in Fig. 3.20 occur at $|\omega - \omega_L| = \pi v/w$.

3.14 A quiz on fluorescence and light scattering

Here we present a collection of conceptual questions that are designed to test one's understanding of several key ideas in spontaneous emission and scattering, and help develop intuition in these subjects. In order to minimize possible confusion, we attempt to clearly specify a physical situation to which the question pertains, although the concept illustrated by the question may be of a more general nature. Testing these questions on our colleagues (and ourselves) has convinced us that some of the questions may not be as trivial as might seem at first glance.

(a) A free two-level atom at rest in its ground state is irradiated by a pulse of off-resonant radiation with a Gaussian temporal profile. The light is nearly