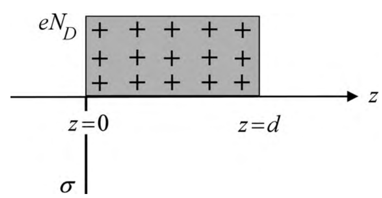
Phys 232 Problem Set 5

Released: 4/01/2020
Due: 4/15/2020

References: David Tong's Electromagnetism notes, Zangwill Chapters 4.6, 6, 17

Problem 1

A semiconductor with permittivity ϵ occupies the space $z \geq 0$. One "dopes" such a semiconductor by implanting neutral, foreign atoms with uniform density N_D in the near-surface region $0 \leq z \leq d$. Assume that one electron from each dopant atom ionizes and migrates to the free surface of the semiconductor. The final result (illustrated by the diagram) is a region with uniform positive charge density eN_D and a layer of negative charge with density σ localized at $z = 0$.



(a) Find and sketch the electric field $E_+(z)$ at every point in space produced by the volume charge.

(b) Find σ and the electric field $E_-(z)$ produced by σ. Sketch E_- on the same graph used to sketch E_+ in part

(c) Sketch the total electric field and check that your graph is consistent with integrating Gauss' law from $z = -\infty$ to $z = \infty$.

Problem 2

Application 6.3 in Zangwill modeled a meson (a quark-antiquark pair) as a finite dipole placed at the center of a spherical cavity with radius R and unit dielectric constant scooped out of an infinite medium with dielectric constant $\kappa \to 0$. For this problem, we replace the finite dipole by a point dipole $\boldsymbol{p}$.

(a) Find $\boldsymbol{D}$ and $\boldsymbol{E}$ everywhere for finite κ.

(b) Confirm the statements made in the Application regarding $\boldsymbol{D}$ and U_E when $\kappa = 0$. Assume a cutoff distance $a \ll R$ to simulate the size of the original dipole.

Problem 3

A point source of light is embedded near the flat surface of a dielectric with index of refraction n. Treat the emitted light as a collection of plane waves (light rays) that propagate isotropically away from the source. Find the fraction of light rays that can refract out of the dielectric into the vacuum space above.

Problem 4

The optical properties of a remarkable class of materials called topological insulators (TI) are captured by constitutive relations which involve the fine structure constant, $\alpha = (e^2/\hbar c)/(4\pi\epsilon_0)$. With $\alpha_0 = \alpha\sqrt{\epsilon_0/\mu_0}$, the relations are

$$\boldsymbol{D} = \epsilon\boldsymbol{E} - \alpha_0\boldsymbol{B} \tag{1}$$

$$\boldsymbol{H} = \frac{\boldsymbol{B}}{\mu} + \alpha_0\boldsymbol{E} \tag{2}$$

(a) Begin with the Maxwell equations in matter with no free charge or current. Show that a monochromatic plane wave of $(\boldsymbol{E}, \boldsymbol{B})$ is a solution of these equations for a TI and find the wave speed.

(b) A plane wave with linear polarization impinges at normal incidence on the flat surface of a TI. Show that the transmitted wave remains linearly polarized with its electric field rotated by an angle θ_F. This is called Faraday rotation of the plane of polarization.

(c) Show that the reflected wave remains linearly polarized with its electric field rotated by an angle θ_K. This is called Kerr rotation of the plane of polarization.

Problem 5

Consider a setup in which the region $y < 0$ is vacuum and the region $y > 0$ is filled with material where $\mu = \mu_0$ and $D_{ij} = \epsilon_{ij}E_j$. Let α, β, and γ be real numbers and take the dielectric matrix as

$$\boldsymbol{\epsilon} = \epsilon_0 \begin{pmatrix} \alpha & i\beta & 0 \\ -i\beta & \alpha & 0 \\ 0 & 0 & \gamma \end{pmatrix} \tag{3}$$

(a) Write out the electric field everywhere if a wave incident from the vacuum is $\boldsymbol{E} = E_0\hat{\boldsymbol{x}}e^{i\omega(\frac{y}{c}-t)}$.

(b) Repeat part (a) if the incident field is $\boldsymbol{E} = E_0\frac{\hat{\boldsymbol{x}}+\hat{\boldsymbol{z}}}{\sqrt{2}}e^{i\omega(\frac{y}{c}-t)}$.