Chem 163, 2022
HW #8
Due 9 am 11/29/2022

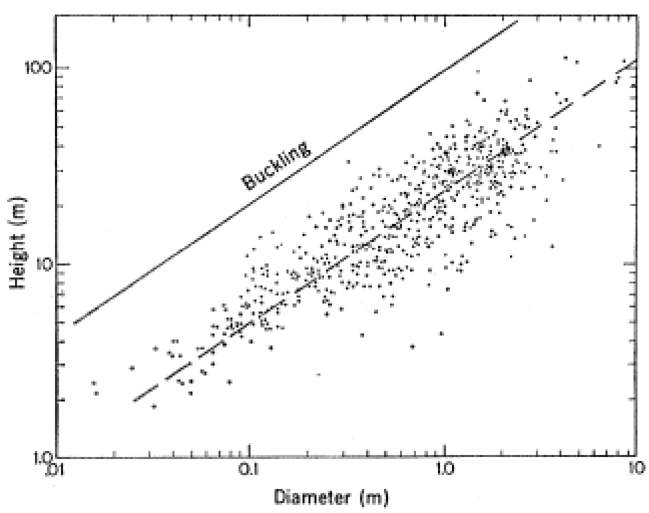
1) Allometric scaling.
This problem is inspired by:
McMahon, Thomas. "Size and shape in biology." *Science* 179.4079 (1973): 1201-1204.

In class we derived that a cylindrical rod under a compressive force will buckle if the force exceeds a critical value called the Euler buckling force:

$$F_b = \frac{\pi^2 YI}{(KL)^2}$$

where *Y* is the Young's modulus, *I* is the moment area of inertia, and *L* is the length. *K* is a numerical factor of order 1 that depends on the boundary conditions at the ends of the rod. This formula is strictly accurate only for forces applied to the ends of the rod, but the scaling is correct for axial body forces applied along the length as well.

The plot below shows data on the heights and diameters of 576 individual North American trees of many different species.



The dotted fit has a slope of 2/3. Model a tree as a cylinder of height *L*, diameter *d*, density ρ, and Young's modulus *Y*. Derive the following scaling relationship:

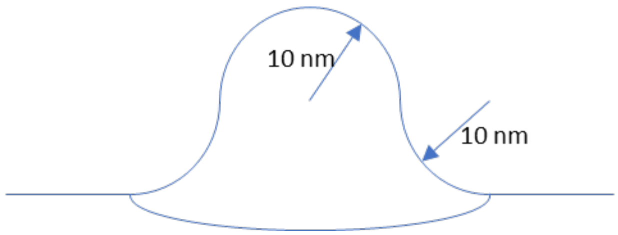
$$L \sim \left(\frac{Y}{\rho}\right)^{1/3} d^{2/3}.$$

2) Mechano-sensitive ion channels
Refer to this recent paper from the lab Rod MacKinnon (2003 Nobel Laureate for his work on ion channel structures):

Guo, Yusong R., and Roderick MacKinnon. "Structure-based membrane dome mechanism for Piezo mechanosensitivity." *Elife* 6 (2017): e33660.
https://cdn.elifesciences.org/articles/33660/elife-33660-v2.pdf

This problem is based on the Discussion of this paper, Figure 7, and Figure 7-supplement 1.

Suppose the Piezo1 protein introduces a deformation into the membrane comprised of a hemispherical cap of radius 10 nm, and then a toroidal joint to the planar membrane, also of radius 10 nm:



A) Calculate the surface area of this deformation. Suppose there is a tension σ in the membrane. How much would the mechanical energy of the membrane decrease if the membrane underwent a transition to a flat state, i.e. where the same quantity of membrane shown above was in a flat disk?

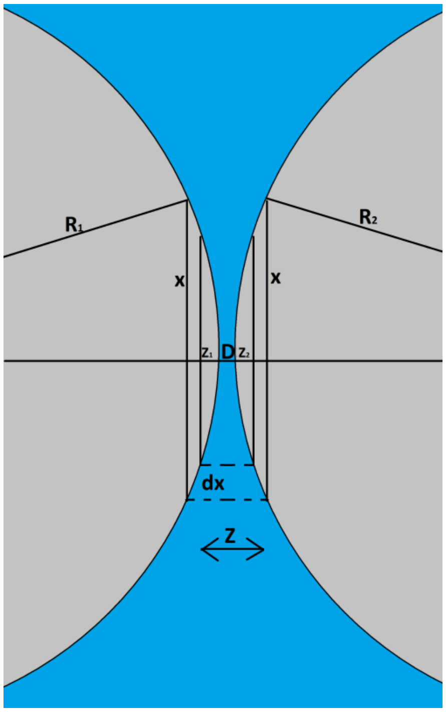
B) Calculate the elastic energy of this conformation.

C) Explain, in words, the formula in Eq. 3 and the formula for the curves plotted in Fig. 7d.

3) Laplace-Young relation. Phillips problem 11.4 (see scanned text on website)

4) Derjaguin approximation.
The Derjaguin approximation provides a means to estimate the force between bodies of different shapes. Consider the geometry shown below, where two spheres of radii R_1 and R_2 are separated by a distance $D \ll R_1, R_2$. Assume that the interaction between the spheres falls off over a much smaller distance than either R_1 or R_2. Let $W(z)$ be the interaction potential (energy per unit area) for flat surfaces separated by distance z. Treat the interaction between the spheres as a sum of planar interactions between concentric thin annuli (see Figure). Prove that:

$$F(D) \approx 2\pi \left(\frac{R_1 R_2}{R_1 + R_2}\right) W(D).$$



How does the force between a sphere and a wall compare to the force between two spheres of equal radius?