Announcements

Office hours this week:
4-5 PM on Tuesday the 13th (right after section!)

7-8:30 PM on Wednesday the 14th, virtual at https://harvard.zoom.us/j/98197473635.

or by appointment.

Section next week:
No section :(((. Section problems and solutions will be posted on Canvas.

Cool paper(s) I'm reading:
https://rdcu.be/cVi10
Multiplexed, single-molecule, epigenetic analysis of plasma-isolated nucleosomes for cancer diagnostics: Cool application of single-molecule fluorescence imaging to cancer diagnostics.

Problem 1: Diffusion-limited off rates

In class, we derived an expression for the maximum reaction rate, assuming that diffusion is the sole limiting factor. We can perform a similar calculation to calculate the maximum possible off rate.

a) First, let's remind ourselves of the solution to the diffusion equation in 3 dimensions from a spherical point source. Let the concentration infinitely far from the point source decay to 0. The steady-state diffusion equation in spherical coordinates with spherical symmetry is:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dc}{dr}\right) = 0. \tag{1}$$

Solve this equation. There will be an unknown constant in your equation—don't worry about it for this part.

b) Now, consider a spherical particle A (immobile in this reference frame) initially bound to N spherical particles B. To model the steady-state flux away from the surface of A, we define a spherical shell of radius R which is just big enough to contain the N particles of B. We then release the B particles and whenever a particle of B diffuses outside this shell, we replace it by a new particle of B at radius R. Using this information, determine the constant left in your equation from part (a).

c) Now using Fick's first law, determine the flow of particle B away from the surface of particle A. Report your answer as a rate. Use the Stokes-Einstein relation to eliminate D from your result.

d) Using your expression derived in (c), estimate the maximal off-rate for a pair of typical proteins. Feel free to look up any constants you may need and to make a reasonable guess as to the typical size of a protein.

Problem 2: Steady-state diffusion

Suppose that in 1D, two bacteria are separated by a distance L, and a nutrient source is placed at a distance x_0 from one of the bacteria (and $L - x_0$ from the other). Assuming the source continuously produces nutrients while the bacteria continuously absorb them, then a steady state will be reached.
Here, we will determine what fraction of the nutrients will reach each of the two bacteria, depending on the source's position. This is an analytical version of a problem which you solved numerically on your HW.

a) Treat this as a 1D diffusion problem (see above) with absorbing boundaries at $x = 0$ and $x = L$. Assuming the concentration is always c_0 at the source ($x = x_0$), solve for the steady state concentration of nutrients as a function of x. Hint: Solve the steady-state diffusion equation separately to the left and to the right of $x = x_0$, then match boundary conditions at x_0.

b) Compute the flux at $x = 0$ and $x = L$. From here, compute the fraction of nutrients absorbed per unit time by each of the two bacteria. How does this depend on x_0?

Problem 3: Time-dependent diffusion in a pipe

Let's now consider time-dependent diffusion in a simple, 1D case. Say that we have a column of solution in which some chemical species is at concentration c_0. At time $t = 0$, we expose this column to another one at concentration 0. Let the boundary between the two columns be at $x = 0$.
a) Write down a set of boundary conditions for $t = 0$.

b) The time-dependent diffusion equation here has the solution:

$$C(x,t) = \frac{c_0}{2}\left[1 + erf\frac{x}{(4Dt)^{1/2}}\right], \tag{2}$$

where erf(x) is the error function:

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du, \tag{3}$$

which can be evaluated numerically in Matlab using the erf() function. Working in units in which $D = 1$, plot the concentration profile as a function of x for the following times: $t = 1, 4, 16, 64$. Comment on your results.