Announcements

Office hours this week:
No office hours on Tuesday the 20th

7-9:30 PM on Wednesday the 21st, virtual at https://harvard.zoom.us/j/98197473635. Note the extra hour this time!

or by appointment.

Section next week:
In-person on September 27th at 3 pm in M217.

Problem 1: Autocorrelation functions

Consider a particle subject to fluctuating forces due to collisions with its environment. Let's treat this as a simple 1D system. At time $t = 0$, the particle is subject to a force F whose value is drawn from a Gaussian distribution centered at zero with variance σ^2.

Suppose that this force continues until some later time t', at which point the particle is subjected to a different force F' (due to another collision) drawn independently from the same Gaussian. The *time* t' at which this force changes is drawn from an exponential distribution: $p(t) = \frac{1}{\tau_0} \exp(-t/\tau_0)$

a) Show that the force autocorrelation function, defined as $C(\tau) = \langle F(0)F(\tau) \rangle$, is given by $\sigma^2 e^{-\tau/\tau_0}$.

b) In class, we argued that $C(\tau) = 2\gamma k_B T \delta(\tau)$ where $\delta(\tau)$ is a delta function centered at zero. Assuming that τ is very small (so that a rapidly decaying exponential behaves like a delta function), relate the result from part **a** with the result from class by writing σ^2 in terms of γ and $k_B T$

Problem 2: The method of images

I got some questions last week about the method of images. This problem will guide you through the rationale behind this method and then provide an example application. We'll start by proving some properties of solutions to Laplace's equation (which will also be true of the steady state diffusion equation).

a) For this problem, consider the 1D steady state diffusion equation:

$$\frac{d^2c}{dx^2} = 0. \tag{1}$$

Show that solutions to this equation permit no local maxima or minima. All extrema must occur at the boundaries.

This result also holds in 2D and in 3D. You can either take my word for it or extend your method in (a) to higher dimensions.

b) Using the result from part (a), show that solutions that satisfy the steady state diffusion equation for some set of boundary conditions are unique. *Hint:* assume that there are two distinct solutions to the equation and show that that this leads to a contradiction.

The consequences of this uniqueness theorem are profound: For any set of boundary conditions for a given region of space, there is only one function that satisfies the steady state diffusion equation. In the method of images, we leverage this property by replacing a challenging problem with a simple one that has the same boundary conditions. Here's an example:

c) We place a diffusive source of radius R and fixed concentration c_0 at a distance z_0 to the left of an infinite reflecting plane. Calculate the concentration profile in the region to the left of the reflecting plane. *Hint:* consider the scenario with a diffusive source of concentration c_0 at a distance of $2z_0$ to the right of the source (and no reflecting plane). Compare the boundary conditions in these two scenarios.